University of Banja Luka I Faculty of Architecture, Civil Engineering and Geodesy
ISSN 2712-0570 I UDC 72

AGG+

2023_11(1)

AGG+ Journal for Architecture, Civil Engineering, Geodesy and Related Scientific Fields
АГГ+ часопис за архитектуру, грађевинарство, геодезију и сродне научне области

064-082

Categorisation I Original scientific paper
DOI I 10.61892/AGG20230164
UDC I 766:659.113.1
Paper received I 28/02/2023
Paper accepted I 28/05/2023

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PRINCIPLES OF DESIGN, MATERIALISATION, AND OPTIMISATION OF THE STRUT-TYPE HYBRID SYSTEMS

ABSTRACT

The strut-type hybrid system can be made in different geometric shapes, which are affected by: the number and arrangement of struts, the shape and position of the cable in relation to the girder, and the size and shape of the cross-section of the girder. When choosing a system, all the listed parameters can vary geometrically, which has consequences on the behaviour of the girder when carrying the load by directly affecting the change in the stiffness of the system. In addition to the geometric parameters, the stiffness of the hybrid system is affected by variations in the properties of the incorporated materials and their mutual relations. In this paper, based on a detailed parametric analysis, the principles of design, the choice of materialisation, and the possibility of further optimisation of the persistent-type hybrid systems are given, with the aim of additionally increasing the load capacity and reducing the deformability. The results of this research are presented in the form of general expressions and diagrams, which can be applied with sufficient accuracy in practice when choosing the form and materialisation of the hybrid system, as well as the possibility of further optimisation of the system by applying pre-stressing. Using the example of external pre-stressing of a glued laminated timber girder, with the assumption of ensuring the lateral stability of the system, the results of the analysis of the behaviour of such a system under load in real conditions, i.e. the influence of the environment and changes in material properties, are given.

Keywords: *hybrid system, behaviour, geometric stiffness, materialisation, pre-stressing*

1. INTRODUCTION

The development of modern lightweight load-bearing structures requires a new approach to design, which involves looking at the behaviour of the entire system. The main task is to realize the mutual dependence of the geometric arrangement of the load-bearing structure's elements and form, whereby the load-bearing structure should meet the conditions of efficiency, usability, and cost-effectiveness. Structural elements are grouped according to their load transfer behaviour in a geometric order that ensures sufficient load capacity and functionality of the structure while optimising material consumption.

The efficiency of the load-bearing structure is expressed by the ratio of the associated dead and live load. In modern load-bearing structures, this ratio has been significantly reduced owing to the knowledge and more efficient application of the properties of modern high-strength materials, in combination with structural systems predominantly stressed in tension.

The usability of the load-bearing structure is characterized by its form and deformability. Practically, the usability of the system represents its adaptability to load changes in operating conditions, which, in addition to the value of performance and maintenance, as well as the possibility of protecting the material and structure, contributes to the assessment of the cost-effectiveness of the load-bearing structure.

The existing traditional calculation methods point to the constructive possibilities of the load-bearing structure, mainly reduced to elements defined by the material. Modern lightweight bearing systems can be analysed from the aspect of their adaptability to changes regardless of the type of material, which provides opportunities for the development of systems with high utility values while ensuring material savings. Given the fact that such structures, due to their own or implied tensile stress, achieve stability during compressive or bending stress, they also have optimal energy consumption during load transfer.

The solution to the set requirements is the development of hybrid load-bearing systems, which provide the possibility of changing form and behaviour, as well as increasing the load-bearing capacity of the "basic" systems they are derived from.

2. FROM THE CONTINUUM TO THE ROD SYSTEM

The formation of rod structures from a continuum with certain boundary conditions, support conditions, and associated loads, with the aim of optimising material consumption, represents the process of forming stress fields and associated directions of action of the main stresses. Within a continuum, the direction of forces is determined by the direction of the greatest resistance, which at the same time represents the direction of the least deformation. If the flow directions of the continuum forces are developed into a network of rods, with precisely determined joints, minimum energy consumption for load transfer is ensured.

Load-bearing systems made of linear elements can be made in an unlimited number of geometric forms with different patterns of mechanical behaviour. Line elements are loaded with axial tension or pressure forces, and the mechanical behaviour of the system is directly conditioned by the number of connected elements in one joint. The basic interdependence of the behaviour of rod-bearing systems on the number of connected elements in one joint is that increasing the number of rods in a joint reduces the geometric deformability of the system. The internal structure of such systems is determined by the direction, arrangement and angle of load action, and the support conditions and stiffness of individual elements significantly influence the mechanical behaviour of the system.

Rod-bearing systems formed according to the principle of optimal energy consumption during load transfer simultaneously meet the conditions of usability and cost-effectiveness.

3. STRUT-TYPE HYBRID GIRDERS – PRINCIPLES OF SHAPING

If we take a beam of homogeneous and isotropic material as an example of a continuum, by applying the described principle of minimum energy consumption, we can form optimal hybrid load-bearing systems. Image of beam stress trajectories, loaded as in (Figure 1.), allows the interpretation of its bearing capacity, where the system of pressed and tensioned arches are mutually kept in balance. The trajectories of the main stresses, shown in the figure, intersect each other at an angle of 90°, and the stress magnitudes along one trajectory are variable. The principal stresses intersect the neutral axis at an angle of 45° and both principal stresses are equal in value to t. The forces in the arches are equal to 0 at the supports and have their maximum value at the crown. The horizontal components of these forces balance the moment, and the vertical forces balance the transverse force in the cross-section. For the illustration, (Figure 1.a.) shows changes in normal and shear stresses according to the height of a rectangular cross-section stressed by bending moment M and positive transverse force V [1].

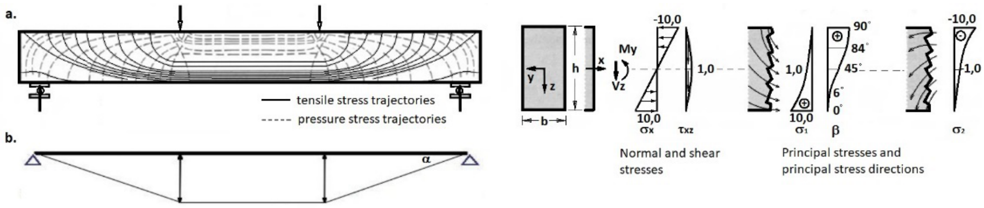


Figure 1. *a. Stress trajectories of a beam made of homogeneous and isotropic material loaded in bending, b. replacing the strut-type hybrid system [1] - adapted by the author*

The strut-type hybrid system is created by the constructive unification of two "basic" systems, i.e. a girder that has bending stiffness and a cable that has stretching stiffness, with common support points at the ends of the girder (Figure 1.b.). Their constructive unification is provided by verticals of infinite stiffness, hinged to a rigid girder. They are advantageous in cases of large spans or loads where elements with bending stiffness would require a disproportionately large consumption of material as the span increases.

Individually, these two "basic" systems have completely different load-bearing qualities, which directly depend on their behaviour under load. Changing the geometric and material characteristics of individual systems causes a change in their behaviour, but the systems retain the same load-bearing quality. By constructively unifying "basic" systems, grouping them within the hybrid system in proper geometric order according to the behaviour during load transfer, any change in form, a cross-section of elements, or choice of materials, significantly affect the load-bearing quality of that system.

3.1. DETERMINATION OF EFFECTIVE CABLE SHAPE

The cable within the hybrid system represents an affine structure, strained exclusively in tension. This element is mechanically completely used in the cross-section. In addition to the task of taking over the tensioning force, the cable within the hybrid system has the function of geometrically adjusting the system to the given load. The geometry of the deformable cable varies, due to the lack of bending stiffness, adapting to the load transferred through the rigid verticals (struts). A mixed (hybrid) system is formed by overlapping with a bending-rigid element. In this way, the system gets rigid components, which ensures the limitation of deformations of the mixed system.

When finding the geometric shape of a cable, it is necessary to distinguish between a deformable cable (chain) and a rigid cable (cord). It is necessary to emphasize that the cord, in addition to the tensile stiffness, has a certain bending stiffness, which has an impact on the different initial shapes of the cord without external load.

The cord line differs from the line of a deformable cable (chain) because the cord has a certain bending stiffness, and the cord line has a larger radius of curvature in the crown area than the chain line. It means that there is a difference in the calculation of the force in the support of cord F_S^H (differential equation of force change) and chain F_L^H (cord geometry and application of Hooke's law), as well as the cord arrow f_S and chain f_L, all depending on the relationship between the length of the cord and the span of the supports (*L/l*). According to [2], the calculation of the cord line with its own weight can be taken simplified by the equation of the parabola of the second order up to the ratio L/l~1,20.

One of the ways of finding the correct geometrical shape of the cable within the hybrid system is to set the condition that, for the shape of the external load on the system, provided relatively affine transmission loads through the struts on the cable. If it is additionally assumed that the girder rests on vertically immovable intermediate supports, a replacement system of continuous girder is obtained (Figure 2.b.).

Thus determined magnitudes of the reactions of the intermediate supports for the associated load on the girder provide a reduction in stress on the girder. The geometry of the cable is chosen from the condition that the normal forces in the struts are in the same ratio as the reactions of the intermediate supports (Figure 2.c.). The cable then cooperates without elastic deformations.

The force in the cable is determined from the balance of the joint, and from the ratio of the forces in the struts, the arrow ratio (*f'/f*) is obtained, where f is an arbitrary quantity.

$$\frac{R_V'}{R_V} = \frac{m \cdot ql}{n \cdot ql} = \frac{F_V'}{F_V} = \frac{2 \cdot f' - f}{2 \cdot \left(f - f'\right)} \rightarrow \frac{f'}{f} = \frac{m}{n} \tag{1}$$

Where:

m/n – reaction ratio of the intermediate supports of an analogous continuous girder.

If the hybrid system is considered as the sum of two substituting systems, namely as a pure system with bending stiffness and vertically immovable supports loaded with a total external load, and the hybrid system loaded with a substituting load of strut forces, the values of the superimposed bending moments on the girder are smaller compared to the same girder without strut (Figure 2.d.).

The redistribution of the load on the elements of the hybrid system exclusively depends on the stiffness ratio of these two "basic" systems. Thus, the part of the replacement load carried by the rigid element and the part carried by the cable is shown in (Figure 2.e.), where the coefficients of participation in load transfer are k_n and k_s, that is, the stiffness of the girder and cable, respectively. It means that it is united into one system, under the condition of vertical immobility of the intermediate supports $k_n – k_s = 1$, within which the relationship *f/l* figures.

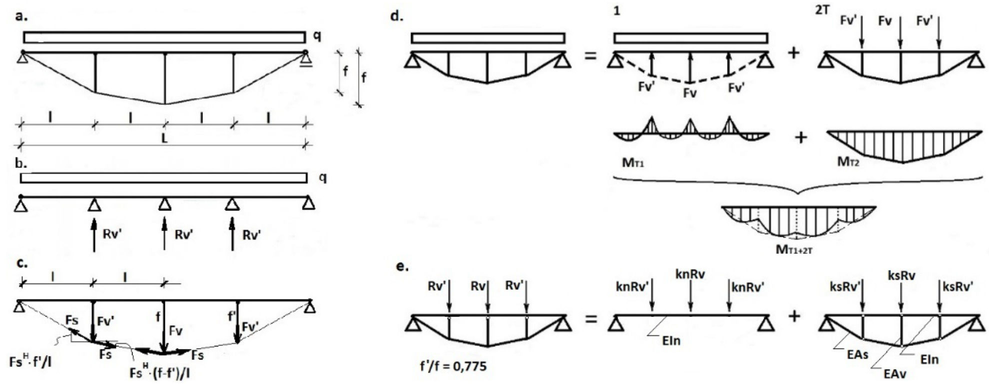


Figure 2. *a. Strut-type hybrid system, b. A replacement model for determining the effective cable shape, c. The value of the forces in the cable and struts for the composition of the cable without elastic deformations, d. Approximate load redistribution in the hybrid system, e. Redistribution of the load depending on the stiffness of the system elements [3] - **adapted by the author***

Depending on the geometric shape of the cable and its position in relation to the rigid element, it is possible to conclude that by "eliminating" the differences between the geometric shape of the cable and the line of bending moments of the rigid element, we get more economical solutions.

3.2. THE INFLUENCE OF CABLE GEOMETRY ON THE STIFFNESS OF THE HYBRID SYSTEM

The stiffness of the hybrid system consists of partial stiffnesses, namely: stiffness to elastic deformations of individual elements of the hybrid, and stiffness conditioned by the geometric shape of the cable in relation to receiving the load.

According to [4], by Second-order Theory, it is possible to obtain approximate formulas for the dependence of the force in the cable and displacement depending on the relevant parameters (geometry, stiffness, load, and method of support). This approximate solution was obtained on a planar system of suspended, uniformly curved, cables with fixed supports, and can be used to derive general, valid statements for transferring load by cables (individually and in networks). Approximate expressions according to the Second-order Theory can also be applied for strut-type hybrid girders, and for finding displacements in the longitudinal direction of the cable.

The length of the cable L is given by the general expression for the length of the curve:

$$L=\int_0^l \sqrt{1+y'(x)^2}\,dx \tag{2}$$

The influence of elastic deformability results from the change in the length of the cable due to the action of force according to the First-order Theory and is:

$$\Delta L = \frac{F_{S,p}^{H}}{E \cdot A} \int_0^l \left(1 + y'(x)^2\right) dx \tag{3}$$

Where:

- $F_{S,p}^{H}$- horizontal component of the force in the cable,
- ΔL - cable length change.

For relatively small deformations, it can be stated that *y(x)* only increases with the added load and that the influence of cable elongation on the change in the shape of the cable line is such that y(x) differs slightly according to Theory II and Theory I, i.e., these two curves are affine. The change in the ordinate of the cable, $\Delta y(x) = y^{II}(x)- y^{I}(x)$, where $\Delta y(x) \ll y^{I}(x)$ represents the cause of the reduction of the force in the cable, which can be calculated using the Eq. (3) for $F_{S,P}^{H,I}$, and using Eq. (2) to obtain a new cable line. The iterative process leads to more accurate solutions.

For relatively small changes in geometry, the iteration procedure can be reduced to the introduction of an unknown force according to Theory II in Eq. (3). The unknown quantities in relation to the output values according to Theory I are ΔF_S^H, ΔL and the change of the arrow Δf, where:

- ΔF_S^H - is defined as the change in force in the cable from a live load (it is assumed that the force in the cable from the dead load is equal to the force from the dead load according to Theory I.),
- ΔL - elastic elongation of the cable with a horizontal force according to Theory II (assuming $\Delta y(x)= yI(x)$),
- Δf - follows from the elongation of the cord as a change in the characteristic ordinate in *l/2*.

If a geometric connection is established between ΔL and Δf, two mutually independent equations are obtained that provide solutions for:

- ΔF_S^H due to the change of Δf

$$\Delta F_S^H = -\frac{\Delta f}{f} \cdot F_S^{HI} \tag{4}$$

- i.e., the value Δf from ΔL, using Eq. (3) with the horizontal force according to Theory II:

$$\Delta f = \frac{F_S^{HII}}{E \cdot A} \cdot f \cdot G$$
$$G = \frac{1}{f} \cdot \frac{\Delta f}{\Delta L} \cdot \int_o^l \left(1 + y'(x)^2\right) dx \tag{5}$$

By introducing the characteristic geometric value *G*, which represents the relation between the change of the arrow and the elongation of the cable, from the previous Equations (4) and (5), it is possible to directly obtain the value of the force in the cable according to Theory II, namely:

$$\Delta F_S^H = F_{S,p}^{H\,II} - F_{S,p}^{H\,I} = -\frac{F_{S,p}^{H\,II}}{E\cdot A}\cdot G\cdot F_S^{HI}$$
$$F_{S,p}^{H\,II} = \frac{F_{S,p}^{H\,I}}{1+\dfrac{G\cdot F_S^{HI}}{E\cdot A}} = \frac{F_{S,p}^{H\,I}}{1+K} = \beta\cdot F_{S,p}^{H\,I} \qquad (6)$$

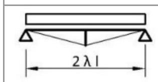
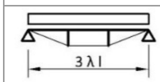
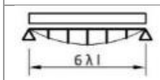
Where:

$$K = \frac{F_S^{HI}\cdot G}{E\cdot A}\ ;\ \beta = \frac{1}{1+K} \qquad (7)$$

On the basis of Equations (6) and (7), the influence according to Theory II β can be determined, and for the known line *y'(x)* of the undeformed cable, the total horizontal force in the cable F_S^H, the geometric characteristic values *G* and the tensile stiffness of the cable *EA*. *K* and *G* are directly proportional values and are valid for all forms of cords as characteristic parameters depending on f^2/l^2.

The characteristic geometric value *G* represents the influence of the cable's geometry in a closed form on the stiffness of the system. Using this value, the geometric stiffness of the hybrid system can be determined with sufficient accuracy considering the characteristic form of the cable, i.e. a unique image of the load transfer to the cable of the system (Table 1). The geometric characteristic G is applied, in addition to the calculation of geometric stiffness parameters, in the calculation of cross-sectional forces and deformations according to the higher order theory.

Table 1. *Geometric stiffness of the characteristic cable forms of the hybrid system [5]*

Strut-type hybrid system	Δf	$K_{G,S}$
 single*	$\frac{q\cdot l^2}{8\cdot f\cdot E_S\cdot A_S}\cdot f\cdot\frac{l^2}{4\cdot f^2\cdot\cos^3\alpha}$	$32\cdot f^2\cdot\cos^3\alpha\cdot E_S\cdot A_S$
 double**	$\frac{q\cdot l^2}{8\cdot f\cdot E_S\cdot A_S}\cdot f\cdot\frac{l^2\cdot(1+\cos^3\alpha)}{9\cdot f^2\cdot\cos^3\alpha}$	$\frac{72\cdot E_S\cdot A_S\cdot f^2\cdot\cos^3\alpha}{(1+\cos^3\alpha)}$
 multiple***	$\frac{q\cdot l^2}{8\cdot f\cdot E_S\cdot A_S}\cdot f\cdot\left(1+\frac{3}{16}\cdot\frac{l^2}{f^2}\right)$	$\frac{128}{3}\cdot f^2\cdot E_S\cdot A_S$

Assumptions:
** the new deformed shape of the cable approximately parallel to the initial default shape;*
*** the horizontal component of the cable's elongation at the point of taking the load is ignored;*
**** an approximate expression can be used for the forms where f/l≤0,25, where the influences by Theory II are taken with sufficient accuracy.*

3.3. CHOOSING AN EFFICIENT HYBRID SYSTEM CONSTELLATION

For the analysis of the stiffness ratio of hybrid systems, that is, the efficient constellation of hybrid systems, as well as the constellation suitable for the optimisation of the hybrid system by pre-stressing, it is necessary to consider the participation of the basic systems in the load transfer.

Figure 3. shows the diagrams of the relations of forces in the characteristic cross-sections of the strut-type hybrid girders with different geometric shapes depending on the number of struts, and in relation to the maximum influences on a simple beam, loaded with a uniform load [6].

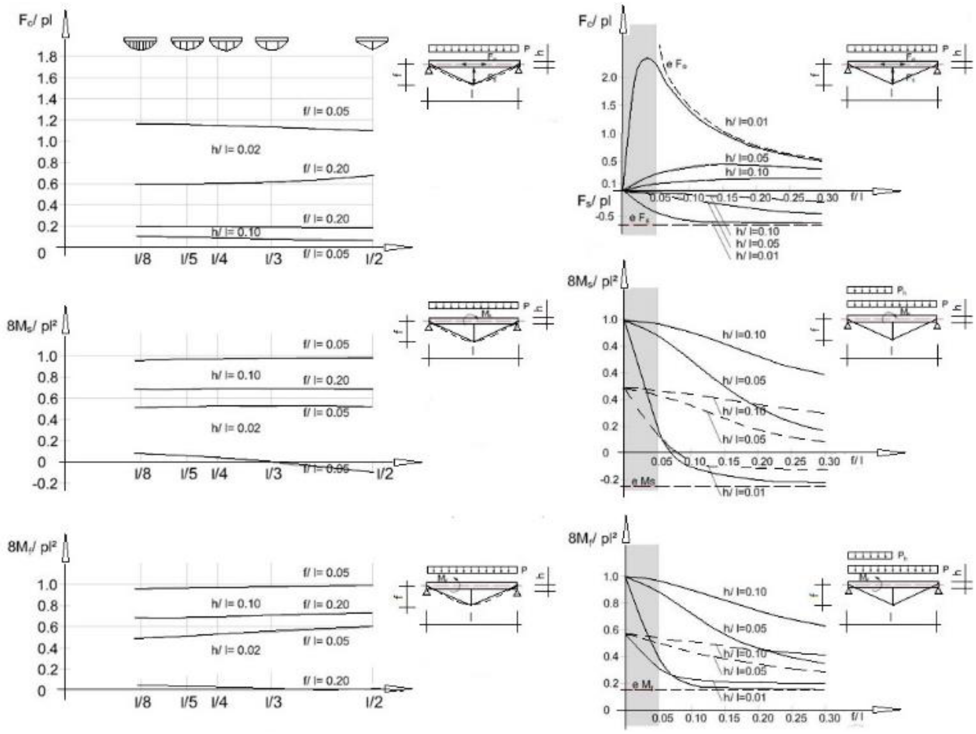


Figure 3. *Relative values of cross-sectional forces on the girder within the hybrid system: a. Dependence of cross-sectional forces on the form of the hybrid system, b. The influence of stiffness changes on the cross-sectional forces of the girder within a single-strut-type hybrid system [6]* ***-adapted by the author***

Based on the diagram (Figure 3.a.), it can be concluded that for rigid girders *(h/l ≥ 0,1)* the number of struts has no influence on load transfer, while for systems with a low girder stiffness *(h/l = 0,02)* and higher geometric stiffness of the cable *(f/l=0,2)*, the differences in the maximum moments are evident, where for a single strut, the moment turns into a negative value for the section above the strut.

Based on the diagram (Figure 3.b**.)**, showing the changes in cross-sectional forces due to changes in the stiffness of the basic systems within the single strut-type hybrid system, it can be concluded that for a small stiffness of the girder *(h/l< 0,05)*, the influence of the cable in the load transfer is of crucial importance. Namely, the image of moments on the girder is similar to the moment diagram of a continuous girder, and every change in the cable force in systems of low geometric stiffness *(f/l=0,05-0,10)* gives a significant change in the normal pressure force in the girder. With an increase in geometric stiffness, the influence of the normal pressure force on the girder decreases, while the bending moments are somewhat smaller than the bending moments of the same basic girder system.

If it is assumed that *f/l = const*. on the hybrid system, as the height of the girder decreases, the moment of values on the girder decreases, and the normal pressure force increases. Thus, it can be concluded that with the application of these systems, taking into account the image of the stress and the type of material used, it is possible to achieve lower consumption of girder material.

Based on the previous considerations, it can be concluded that the geometric height of the hybrid and the height of the girder are the relevant parameters for determining the redistribution of the load in such a hybrid system, and that the image of the load can affect the reduction of the efficiency of the cable and the consumption of material in the girder.

Finally, it can be concluded that the magnitude of the cross-sectional forces is more affected by the reduction of the bending stiffness of the girder than by the increase in the geometric height of the hybrid.

4. MATERIALISATION OF THE HYBRID SYSTEM

Strut-type hybrid systems can be performed in different combinations of materials. In addition to the geometric performance, the behaviour of the hybrid during load transfer is influenced by the differences in the mechanical properties of the materials its elements are made of. The behaviour of the load-bearing systems is characterized by the redistribution of the load and the way it is deformed, which directly depends on the stiffness, i.e. the modulus of elasticity of the incorporated materials.

Figure 4. shows the research of changes in load redistribution for E_S/E_n ratios, where the modulus of elasticity of the cable E_S is a constant value, and the modulus of elasticity of the girder E_n decreases, and in accordance with the applied material of the girder, the areas of change in the stiffness of the hybrid system are indicated.

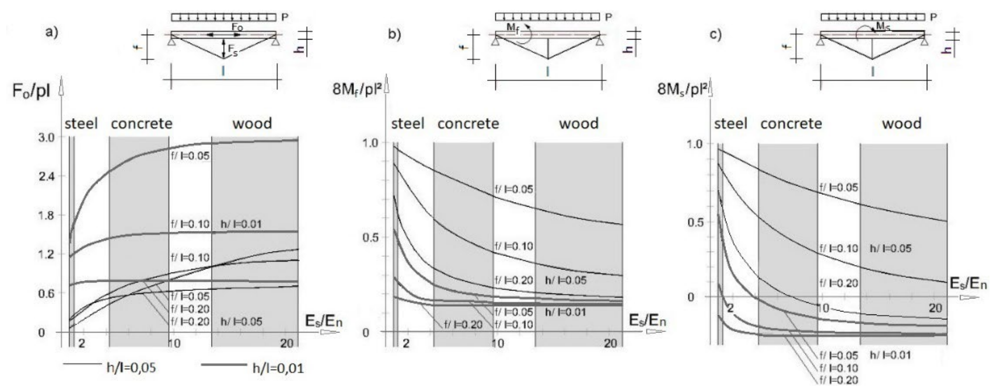


Figure 4. *The influence of the combination of materials on the cross-sectional forces of the hybrid, expressed by the ratio of the modulus of elasticity of the cable and girder: a) normal force in the girder, b) moment in the field, c) moment over the strut [6]* - *adapted by the author*

For hybrid systems in all material combinations shown, with a high-stiffness girder *(h/l)*, the girder takes almost all the load. Systems with a small proportion of geometric stiffness *(f/l=*

0,05) require a larger cross-section of the cable because they have less ability to accept loads than systems with a higher *f/l* ratio.

Systems made with a steel girder, as a result of the high strength and modulus of elasticity of the steel, have a high level of bearing capacity for loads in the plane of the system. Considering the uniformity of the material quality of the girder and cable, in these systems, a more efficient role of the cable is obtained by increasing its cross-section. The sensitivity of the system to lateral buckling, caused by the predominantly low stiffness of the girder in the plane perpendicular to the plane of the system, requires consideration of the spatial stiffening of the system.

In systems with girders made of reinforced concrete or wood, a cable with a small cross-section and any geometric shape contributes to an increase in load capacity compared to a simple girder with the same geometric and mechanical properties. When constructing a hybrid system with a concrete or wooden girder, it is necessary to achieve the transfer of the full horizontal component of the force in the cable to the end section of the girder, without contact between the girder and the cable. Establishing contact of the cable with the concrete girder (monolithic design) or contact with the envelope of the hole in the wooden girder (variability in the dimensions of the wooden girder due to the change in the moisture content of the wood) would cause a change in the value of the force in the cable.

5. OPTIMISATION OF THE HYBRID SYSTEM

According to [7] optimisation of structures implies different degrees of complexity, i.e. optimisation according to the topology of the structure, shape, and dimensions of the cross-section. Optimisation at the level of the topology of the structure implies the determination of the space of the materialized parts of the structure, and the possibility of its optimal solution in terms of omitting material in the space ineffective for receiving loads. At this level of optimisation, conceptual solutions are considered qualitatively, without assumptions about topological connections (number and interaction of structure elements), but only in terms of material distribution and orientation in the structure. Optimisation of the structure shape implies iterative changes of the external and internal boundaries of the distributed material with the aim of meeting the requirements of mechanics. Dimensioning, as the lowest level of a structure optimisation, refers to finding the final dimensions of a given materialized structure.

Previously presented parametric analyses show the results of iterations at the level of optimisation of the shape and dimensioning of the hybrid system from the aspect of carrying capacity. Further consideration of the behaviour of the strut-type hybrid system, applying a suitable materialisation of glued laminated timber-steel, and considering the obtained results of the influence of geometric stiffness, refer to the level of optimisation by dimensioning from the aspect of deformability, considering the possibilities of improving the system by pre-stressing.

5.1. BEHAVIOR OF THE HYBRID SYSTEM WHEN STIFFNESS CHANGES

The behaviour of the hybrid system can be expressed using the strut effect, i.e. the realized force in the strut. For the analysis of the influence of the basic parameters of the hybrid system when transferring the total load, the relative value of the force in the struts of the considered forms of hybrid systems is introduced.

The deformability of the hybrid system can be analysed by observing the hybrid system in the limited cases of the deformability of the individual systems it is composed of [8]. Limiting cases of deformability for a strut-type hybrid system are represented by systems in combinations: a girder with bending stiffness and a deformable cable, and a deformable girder and a cable absolutely rigid in stretching. Observed characteristic cross-sections for establishing the compatibility of deformations are places where the connection of individual basic systems is realized by means of struts. For a system with elements of real stiffness, the behaviour under the load can be described as "deformation of the girder in the section above the strut is corrected by the full pressure force in the strut, which then stretches the cable of real stiffness". The correlation of the deflection of the characteristic girder sections and the cable from the external total load q_{uk} is then the actual value of the pressure force in the support, realized after the deformation of the individual basic systems of the hybrid at the point of connection with the strut. Based on this simplified sequence of load transfer and the appearance of deformations, the exact expressions for determining the value of the actual pressure force in the strut $V_{hyb,eff}$ and the value of the deflection of the hybrid system ω_{hyb} under the action of the external load q_{uk} are obtained.

Parametric analysis was carried out for the limit values of the parameters suitable for the wood-steel hybrid system, namely:

- the ratio of arrow and span: f/l, according to the analysis of suitable constellations for the load-bearing aspect of the hybrid system (limit values $f/l=0,05 \div 0,2$),
- the ratio of the height of the girder and the span: h/l, according to the analogy of the behaviour of the hybrid system with the simple systems of the wooden girder, and the associated orientation values for the height of the cross-section of the wooden girder of the continuous and simple beam systems (limit values $h/l=0,0125 \div 0,055$, in borderline cases of rigid or fully flexible cable),
- the ratio of the cross-sectional area of the girder and the cable: A_n/A_S, according to the condition $V_{hyb,eff}=0$ for different limit values of the previous parameters, and for dry wood, in order to establish the difference in the behaviour of the hybrid girder in real conditions of increasing the moisture percentage of the embedded wood. The upper limit values are determined according to the smallest ratios for different forms of hybrid systems, and for constellations $f/l=0,05$ and $h/l=0,055$, and amount to: $A_n/A_S=122,0$ for a single, $A_n/A_S=394, 01$ for a double, and $A_n/A_S=117,54$ for a multiple strut-type system.

The condition $V_{hyb,eff}=0$, from the expression according to [5] practically means that the strut has no effect on the deformability of the system. In that case, this condition satisfies the equality of partial stiffnesses, the bending girder, and the stretching cable in its geometric form, which practically means that for equal load redistribution between the girder and cable, the hybrid system should be pre-stressed to reduce deformability.

The influence of the change in geometric stiffness was considered by changing the relative force in the strut and for $E_n/E_S=const.$ (for extreme conditions of wood moisture percentage according to EC 5) and variable parameters f/l, h/l, and A_n/A_S (Figure 5.). The characteristic constellations of the system describe the limit values of the relative force in the strut, where the value:

- $V_{hyb,eff}/q_{uk}l = 0$ - represents the equalization of partial stiffnesses (bending girder and stretching cable in a certain geometric form)
- $V_{hyb,eff}/q_{uk}l = 1$ - means that the stiffness of the cable is significantly greater than the bending stiffness of the girder, and the system's load is redistributed to the cable.

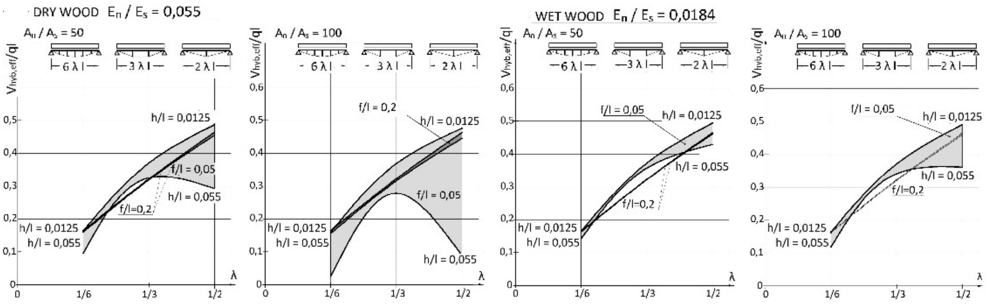


Figure 5. *The magnitude of the relative pressure force in the strut at the point of maximum deflection depending on the geometric stiffness of the hybrid system: a) dry wood, b) wet wood*

Based on the analysis of the parameters shown in (Figure 5.) it is concluded that:

- for small geometric heights of the hybrid system of the considered forms, the greatest strut effect is achieved with the system of a single strut system, and considering the number of struts and the method of load redistribution
- the single strut-type system shows the greatest sensitivity to the change in the geometric height of the hybrid and the bending stiffness of the girder.

The influence of the change in the stiffness of the bending girder, in addition to the change in this partial stiffness expressed by the ratio *h/l*, the change in stiffness with regard to the shape of the cross-section of the girder is especially considered. Two ultimate cases of the shape of the girder cross-section, square and split, were analysed, where the changes in the girder stiffness within the hybrid system are expressed by the ratio $A_n h^2/I_n$.

From the diagram (Figure 6.a.), it can be seen that hybrid systems with a rigid girder and shallow strut can reach the limit stiffness ratios when the behaviour is reduced to the behaviour of a "pure" girder system, so that cable support does not make any sense. For the single strut-type system, where $f/l=0{,}05$ and $h/l=0{,}055$, and for the ratio $A_n/A_S=50$ and $A_n h^2/I_n=4{,}91$, the cable in the system does not have a sufficient role of carrying the load. With the reduction of the cross-sectional area of the cable, i.e. for $A_n/A_S=100$, the limit value for activating the system as a hybrid is for $A_n h^2/I_n=9{,}83$. Thus, for the case of a single-strut-type hybrid system with a split cross-section of the girder, where $A_n h^2/I_n=4$ and $A_n/A_S=50$, the cable makes sense only for geometric heights $f/l \geq 0{,}05559$, i.e. $f/l \geq 0{,}0778$ for $A_n/A_S=100$.

By increasing the percentage of moisture, the bending stiffness of the girder is reduced, and thus the share of the load carried by the cable is increased. Meaning that for hybrid systems with small geometric height and high bending stiffness of the girder, the reduction of the modulus of the girder elasticity (e.g. increasing the moisture of the wood) has a positive effect, that is, it causes the system to activate as a hybrid. For hybrid systems with a small height of the girder cross-section and a small geometric height of the hybrid, the deformability of the unsupported parts of the girder increases additionally, while with a high geometric height of the hybrid, the cable takes almost all the load. For the case of increasing the moisture percentage of the girder of the split cross-section where $A_n h^2/I_n=4$, the cable makes no sense for the single strut-type hybrid systems when: $A_n/A_S=50$ and $f/l \leq 0{,}032$, that is, $A_n/A_S=100$ and $f/l \leq 0{,}046$.

If diagrams (Figures 6.a., Figure 6.b.) are compared, for $A_n/A_S=50 \div 100$ of the same geometric characteristics and the quality of the embedded material, it can be concluded that the double strut-type hybrid system has a more pronounced component of the geometric stiffness of the cable, that is, that in this form the redistribution of the load between the elements of the hybrid is more favourable. This means that the single-strut-type system is more sensitive in terms of changing the stiffness ratio of the system elements.

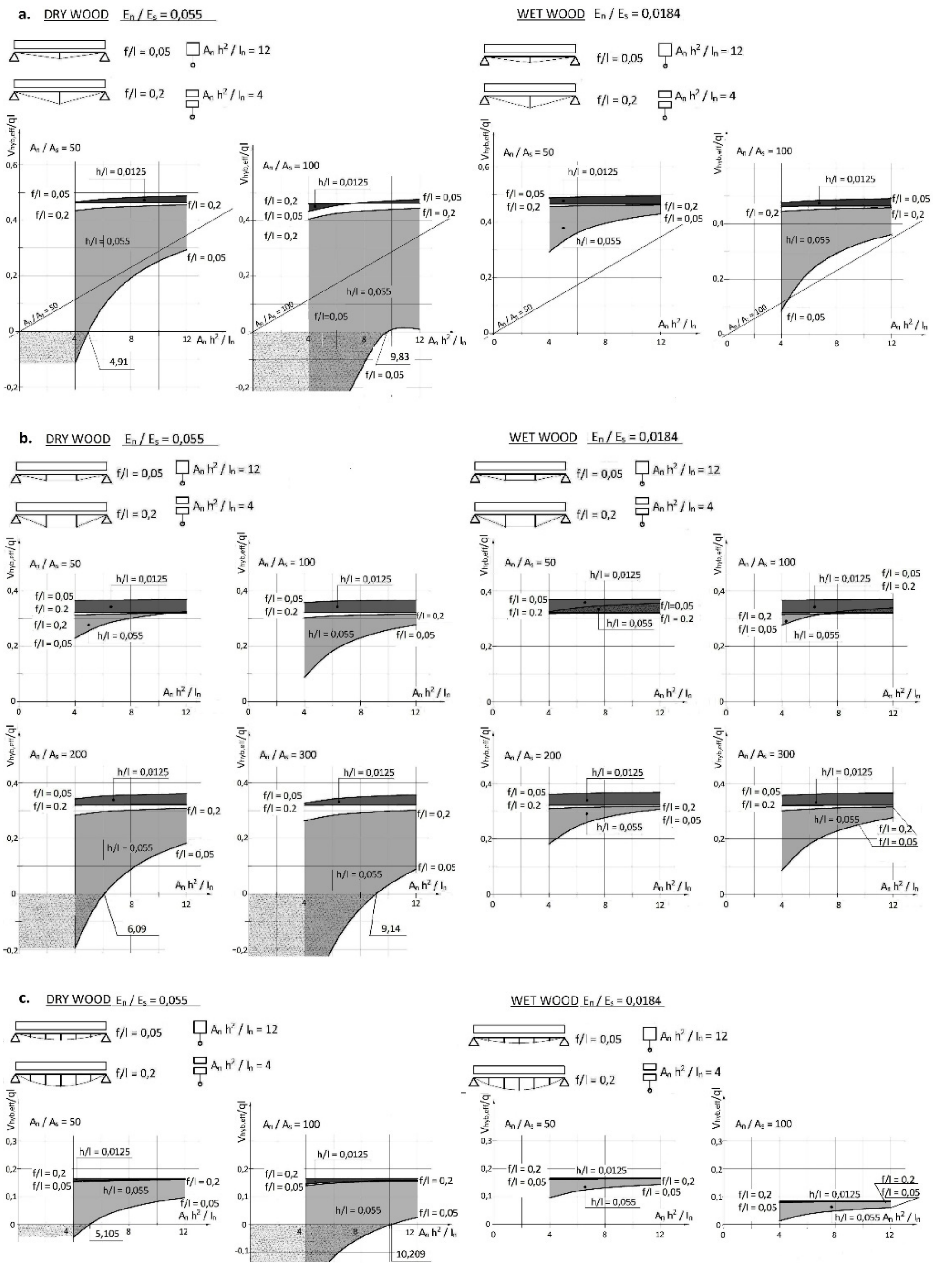


__Figure 6.__ The magnitude of the relative pressure force in the strut of the strut-type hybrid systems depending on the change in the stiffness of the girder with regard to the shape of the cross-section and wood moisture conditions: a. single strut-type system, b. double strut-type system, c. multiple strut-type system [8].

By comparing the behaviour of the system of multiple and single strut-type systems (Figure 6.a., Figure 6.c.), very similar results can be observed. In the case of a multiple strut-type hybrid system, the deformability is less pronounced compared to a single strut, due to the greater number of struts in which the same pressure force values are activated when the cable is in the form of a square parabola. Thus, it can be concluded that for this form of hybrid, it is preferable to use a girder with less bending stiffness compared to the form of a single strut with the same parameter values.

5.2. PRE-STRESSING

The direct influence of pre-stressing on the behaviour of the system during load transfer, in certain individual calculation situations, ensures the reduction of deflection by increasing the stiffness of the system or including the unloaded section zone in receiving forces, increasing the system's carrying capacity and changing the natural frequency of the pre-stressing element.

By pre-introducing the pre-stressing force into the cable of the strut-type hybrid system, the average size of the distributed forces of the system can be positively influenced when the cable is placed in an affine position (geometry) corresponding to the moment line of the girder. In this way, a greater degree of utilization of the section of the element loaded in bending is achieved. Pre-stressing of hybrid systems is one way of increasing the stiffness of the entire system, and it is effective in the case when the hybrid system is composed of linear elements, where the resistance of the cable to the normal force is in balance with the resistance of the girder to bending.

According to [6] depending on the constellation of the system and its materialisation, it is possible to decide the condition for determining the effective pre-stressing force from the point of view of increasing the load capacity of the system. Systems in which the application of pre-stressing is desirable are those in which:

- by activating the cable, it transmits a small value of the pressure force from the action of the external load to the girder, while the bending moments are unique,
- by pre-stressing, it can influence the equalization of bending moment values in sections in the field and above the strut (systems with pronounced stiffness of the split-section girder, as well as "soft" wooden girders within the hybrid system with pronounced geometric stiffness of the cable),
- by pre-stressing, it provides a faster increase in the pressure force in the girder (by a small h/l, where by increasing the geometric height of the hybrid, the pressure forces increase slightly compared to systems with stiffer girders, and the support moments decrease).

From the aspect of the deformability of hybrid systems [9] it can be concluded that the application of pre-stressing makes sense:

- for all forms of hybrid systems, where the unfavourable constellation is the high bending stiffness of the girder and low geometric stiffness of the cable, and for borderline cases when the cable does not have a sufficient role of carrying the load in the system, i.e. it does not make sense without pre-stressing;
- for all forms of hybrid systems, where the pre-stressing of the cable gives it a more dominant partial stiffness, with the aim of reducing the deformability of the overall system.

The size of the pre-stressing force is found with regard to the ratio of partial stiffnesses within the system and the bearing capacity of the characteristic sections, depending on the material incorporated in the hybrid system girder. According to [6] systems with reinforced concrete or wooden girders are suitable for pre-stressing, due to the positive effects of increasing the pressure force in the girder, as well as the redistribution of bending moments in characteristic sections.

The pre-stressing force of the hybrid system with reinforced concrete girder can be found from the conditions of more favourable activation of the concrete part of the cross-section in taking over the cross-sectional forces (according to the predicted degree of pre-stressing), but it is necessary to take into account the changes in the stiffness matrix during elastic deformations (Theory II), as well as a significant change in the stiffness of the girder in Stage II. The change in the cable force in the limit bearing state can be estimated accurately enough when calculating the deformations for the girder in Stage II.

In the example of a hybrid wood-steel girder, the increase in the load capacity of the characteristic cross-sections of the girder is achieved from the condition that by introducing the pre-stressing force, sudden failure of the tensioned edge of the cross-section is delayed by forcing the plasticization of the pressed zone [10]. Failure of the section occurs due to the fully utilized compressive strength as more reliable data, and then by sudden failure of the tensioned zone, due to the increase in tensile stress. In this way, pre-stressing can increase the load-bearing capacity of the cross-section of a wooden beam loaded in bending as if it were wood of a higher quality class than the class of embedded wood. The effective pre-stressing force of the hybrid wood-steel system practically represents the "tightening" of the cable, so that the algebraic sum of the stress on the edges of the girder section from the total force in the cable of the pre-stressed system is equal to the algebraic sum of the compressive strength of the wood and the tension parallel to the fibres [8].

In order to find the magnitude of the effective pre-stressing force in order to reduce the deformability of the hybrid system, it is necessary to observe the system again in the limit cases of the deformability of individual elements, due to the effect of the pre-stressing force on the system. The characteristic observed cross-sections are those in which the connection between the individual basic elements is realized, that is, the cross-sections where the compatibility of the deformations of the individual "basic" systems within the hybrid system can be established. If the hybrid system is loaded only by the pre-stressing force introduced into the cable, the stretching activates a compressive force in the strut that causes vertical deformation of the girder and cable. If these deformations are observed for the ultimate cases of the behaviour of these two basic systems depending on the stiffness, direct dependence of the magnitude of the cable elongation and the deformation of the hybrid system is obtained. By applying this approach, it is possible to obtain accurate expressions for finding the effective size of cable elongation depending on the required deformation of the girder at the point of the strut [9].

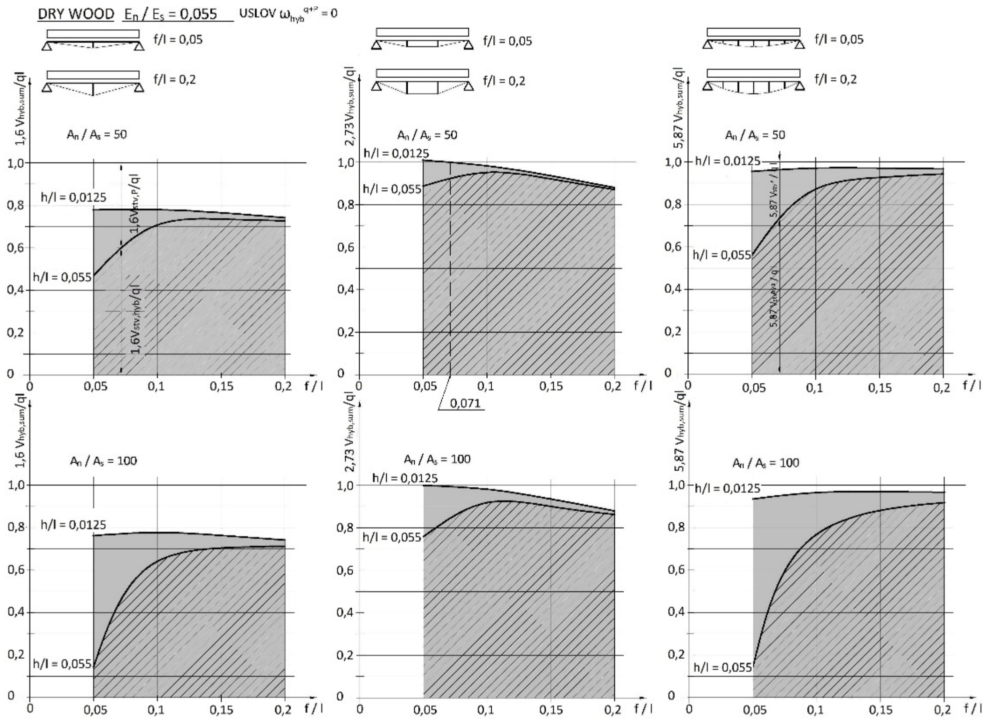


Figure 7. *The magnitude of the relative pressure force in the strut and the required pre-stressing force for the condition ω_{hyb}^{q+P} =0 of hybrid wood-steel systems depending on the change in the stiffness of the girder*

Figure 7. shows the achieved actual values of the forces in the strut of the pre-stressed hybrid system, for the condition of vertical immobility of the characteristic cross-sections at the point of the strut. The exact values of the real force in the strut from the effect of the external load q are determined by derived expressions according to [9]. The relative values of the force in the strut for the external evenly distributed load q are given in Figure 7., where the relative value of the force in the strut to be achieved by pre-stressing is the difference to the full value of 1.

The given diagrams are suitable for practical application in the selection of the strut-type hybrid girders with or without pre-stressing in the combination of glued laminated timber-steel. The value of the force in the strut, realized for the effect of the external load q represents the exact value, and the value of the pre-stressing force must be increased by the expected losses, due to the change in the geometric stiffness of the system during the elastic deformations of the girders of large spans, as well as the effect of friction at the point of the strut.

6. CONCLUSIONS

The choice of a suitable hybrid system constellation represents the process of finding a geometric form, mutual arrangement of load-bearing elements and applied materials, depending on their load transfer capabilities. The expansion of the application of modern lightweight load-bearing structures is conditioned by the development of high-quality modern materials, as well as structure optimisation procedures supported by computer technologies. The results of this work indicate the need to consider and apply theoretical foundations in finding logical solutions, and the presented procedure results in practical expressions and diagrams of sufficient accuracy for practical application when choosing a simple, hybrid, or pre-stressed girder of a certain materialisation.

The behaviour of the strut-type hybrid system depends on the ratio of the partial stiffnesses of the individual elements, and the geometric stiffness of the cable plays a special role in activating the system during load transfer. Changes in the ratio of partial stiffnesses, expressed by the properties of the applied materials or the ratio of geometric sizes, indicate a pronounced sensitivity of the systems in the examined forms, which confirms the need for the practical application of the aforementioned principles of design, materialisation, and optimisation. Using the given expressions and diagrams, the need for iterative adjustment of geometric parameters in the conceptual design phase of these types of structures is excluded.

The parametric analysis of the strut-type hybrid system of wood-steel provided the basic principles of finding constellations of the simple and hybrid girder with and without pre-stressing force, as well as an approximate procedure for finding the pre-stressing force while meeting the conditions of increasing the load capacity and reducing the deformability of the hybrid girder.

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ПРИНЦИПИ ПРОЈЕКТОВАЊА, МАТЕРИЈАЛИЗАЦИЈЕ И ОПТИМИЗАЦИЈЕ ХИБРИДНИХ СИСТЕМА

Сажетак: Хибридни носач типа подупирала могуће је извести различитих геометријских форми, на што утичу: број и распоред потпора, ток силе затезања у ужету, односно облик и положај ужета у односу на носач и облик попречног пресјека носача. Приликом одабира носача сви набројани параметри геометријски могу варирати, што има посљедице на понашање носача при ношењу оптерећења тако што директно утичу на промјену крутости система. Поред геометријских параметара на крутост хибридног система утичу и варијације параметара својстава уграђених материјала и њихови међусобни односи. У овом раду дати су принципи обликовања, избора материјализације и могућности даље оптимизације хибридних носача типа подупирала, а с циљем додатног повећања носивости и смањења деформабилности. Резултати овог истраживања представљени су у облику општих израза и дијаграма, који се са довољном тачношћу могу примјенити у пракси при избору форме и материјализације хибридног система, као и могућности даље оптимизације система примјеном преднапрезања. На примјеру екстерног преднапрезања лијепљеног ламелираног носача, уз претпоставку осигурања бочне стабилности система, дати су резултати анализе понашања оваквог система под оптерећењем у реалним условима, односно утицаја околине и промјене својстава материјала.

Кључне ријечи: *хибридни систем, понашање, геометријска крутост, материјализација, преднапрезање.*