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ACCURACY ANALYSIS OF GNSS PERMANENT STATION COORDINATES USING THE LEAST SQUARE METHOD BY PROCESSING BROADCAST AND PRECISE EPHEMERIDES

ABSTRACT

This paper analyses how broadcast and precise ephemerides in alignment, using the least-squares method, affect the accuracy of coordinates of the newly included permanent stations. It has been shown that in practice, when adjusting networks whose span between points exceeds 50 km, the adjustment procedure should be carried out using the adopted precise ephemerides for a specific observation period. Precise ephemerides are recommended for 30 to 50 km baseline lengths, while adjustments with broadcast ephemerides achieves satisfactory accuracy for shorter lengths. For baseline lengths of 30 to 50 km, the use of precise ephemerides is recommended, and for shorter lengths, adjustment with broadcast ephemerides achieves satisfactory accuracy. This paper analyses statistics and presents standard deviations of horizontal and vertical positions, when broadcast and precise ephemerides are applied.

Key words: *GNSS, least squares method, broadcast ephemerides, precise ephemerides*

АНАЛИЗА ТАЧНОСТИ КООРДИНАТА GNSS ПЕРМАНЕНТНИХ СТАНИЦА ПРИМЈЕНОМ МЕТОДЕ НАЈМАЊИХ КВАДРАТА ОБРАДОМ ЕМИТОВАНИХ И ПРЕЦИЗНИХ ЕФЕМЕРИДА

Апстракт: У овом раду је анализирано на који начин емитоване и прецизне ефемериде у изравнању, примјеном методе најмањих квадрата, утичу на тачност координата новоукључених перманентних станица. Показано је да у пракси приликом изравнања мрежа чији распон између тачака прелази 50 km треба спровести процедуру изравнања примјеном преузетих прецизних ефемерида за одређени период опажања. За дужине базних линија од 30 до 50 km препорука је за примјену прецизних ефемерида, а за мање дужине изравнањем са емитованим ефемеридама постиже се задовољавајућа тачност. Кроз рад је анализирана статистика и представљена су стандардна одступања по положају и висини када се примјењују емитоване и прецизне ефемериде.

Кључне ријечи: GNSS, метод најмањих квадрата, емитоване ефемериде, прецизне ефемериде

1. INTRODUCTION

One of the leading data collection methods today, with extensive applications and possibilities, is the GNSS (Global Navigation Satellite Systems) method. GNSS consists of four main satellite technologies: GPS, GLONASS, Galileo, and BeiDou. Each consists mainly of three segments: a) space segment, b) control segment and c) user segment. Thanks to GNSS technology, end-users now collect and distribute data directly from GNSS tracking stations, known as active control stations (ACS) or real-time kinematics (RTK).

The Differential Global Positioning System (GPS) application is extensive. It is a unique system consisting of a network of fixed stations, a way of transmitting information from fixed stations, and the ability of GPS receivers to receive and process this information.

The network of SRPOS permanent stations consists of 23 evenly placed permanent stations on the Republic of Srpska territory, and it is under the jurisdiction of the Republic Administration for Geodetic and Property-Legal Affairs. Locations for the installation of new GIBL, GIDE, and GITE permanent stations were selected in accordance with the criteria that ensure a smooth operation of the equipment. Three new permanent stations are not under the jurisdiction of the Republic Administration for Geodetic and Property-Legal Affairs.

The choice of basic mathematical processing parameters depends on the source of relative GNSS positioning errors, satellite ephemerides, reference system, tropospheric and ionospheric signal delay.

2. GLOBAL POSITIONING SYSTEM (GPS)

The United States Department of Defense has developed the GPS, an all-time navigation system to meet the needs of the U.S. military and accurately determine their position, speed, and time in a common reference system in each moment anywhere on Earth or near Earth [1].

GNSS signals have very little power, so they are subject to several sources of noise and errors. The range measured by the GNSS receiver is contaminated with errors, which is why it is called a pseudo-range. GNSS errors in a general sense can be classified into errors of satellite origin, receiver errors, errors originating from the middle of the signal movement, and other measurement errors [2].

2.1. SATELLITE EPHEMERIDES

The errors of satellite ephemerides belong to the group of errors of satellite origin. The receivers calculate the satellite position based on the information contained in the navigation message known as satellite ephemerides. These ephemeride parameters are estimated in the control segment and then transmitted to satellites. The satellites broadcast updated ephemerides data every 2 hours. These parameters are estimated using a curve corresponding to the satellite orbit prediction (Figure 1), which leaves residual errors relative to the actual orbit [3].

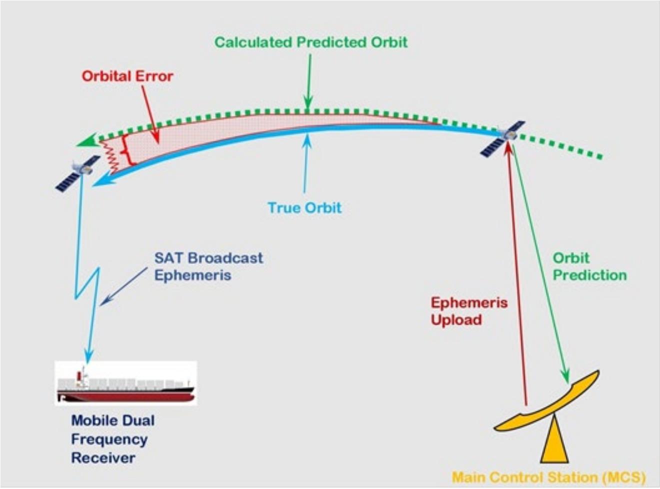

Figure 1. *Simplified representation of a satellite orbit error [4]*

This error source introduces a mean square error LMS (Root Mean Square) of about 2 m [5]. The error can be reduced if global or local area network corrections are available for the satellite position. These corrections are used to improve the correction of broadcast ephemerides and thus to improve the accuracy. More precise ephemerides, available from the International GNSS Service [6], can be used for post-processing if centimetre-level precision is required and when a two-frequency receiver is used.

Highly accurate orbit information is extremely important for determining the position derived from GPS. Broadcast ephemerides generated by the GPS Operational Control Segment (OCS) are available free of charge in real time to users who decode the GPS navigation message and contain data on Kepler elements and their time changes. Although the broadcast ephemerides are accurate enough for most navigation users, high-precision applications need improved orbit information. There are numerous sources of ephemerides that are available online for post-processing.

One of the services that generate ephemerides is the International GNSS Service - IGS [7]. IGS is a key element of the Global Geodetic Surveillance System - GGOS [8] and it fulfils three key roles. The first role is to establish links between SLR observation techniques, VLBI techniques, and DORIS ground beacons. These links are key to the creation of the ITRF International Terrestrial Reference Framework [9]. Another role is to condense and improve the geometric distribution of the global geodetic network, enabling accurate modelling of satellite orbits and clocks, atmospheric behaviour, and terrestrial processes such as neotectonics. The ultimate role is to enable the user segment to access the ITRF, which is increasingly important as the accuracy of publicly available GNSS positioning is improved. From this arises the need for a better understanding of the relationship between the ITRF and national data [10].

For the user to achieve precise positioning, it is necessary to know the orbits and clocks of GNSS satellites. Positioning accuracy is directly affected by errors in satellite paths and clocks. Orbit and clock information, with relatively low accuracy is transmitted via GNSS navigation messages, and other information is provided by IGS. The display of GPS satellite ephemerides, satellite clocks, and stations, as well as their accuracy, latency, continuity, and availability, are shown in Table 1 [11].

Table 1. *Accuracy, latency, continuity, availability, and sampling intervals for IGS orbit and clock products relating to GPS satellite orbits and satellite (sat) and station (stn) clocks as of 2013 [6]*

GPS satellites ephemerides Satellite and station clocks		Sample interval	Accuracy	Latency	Continuity	Availability (%)
Broadcast (for comparison)	Orbits Sat. clocks	-	≈ 100 cm ≈ 5 ns RMS, 2.5 ns σ	Real time	Continuous	99.99
Ultra-rapid (predicted half)	Orbits Sat. clocks	15 min	≈ 5 cm ≈ 3 ns RMS, ≈ 1.5 ns σ	Predicted	4 x daily, at 3 h, 9 h, 15 h, 21 h UTC	95
Ultra-rapid (observed half)	Orbits Sat. clocks	15 min	≈ 3 cm ≈ 150 ps RMS, ≈ 50 ps σ	3 - 9 h	4 x daily, at 3 h, 9 h, 15 h, 21 h UTC	
Rapid	Orbits, Sat. and stn. clocks	15 min 5 min	≈ 2.5 cm ≈ 75 ps RMS, ≈ 25 ps σ	17 - 41 h	daily, at 17 h UTC	95
Final	Orbits, Sat. and stn. clocks	15 min 30 s (Sat.) 5 min (Stn.)	≈ 2 cm 75 ps RMS, 20 ps σ	12 - 18 d	Weekly, Thursday	99
Real-time	Orbits Sat. clocks	5-60 s	≈ 2 cm 300 ps RMS, 120 ps σ	25 s 5 s	Continuous	95

IGS determines several categories of so-called precise ephemerides, some of which are predicted and are available in real time. Final IGS ephemerides, whose accuracy is as much as 2.5 cm, can be freely downloaded from the Internet, unlike broadcast ephemerides, which are only available in a couple of weeks [2].

Since the accuracy of broadcast ephemerides within a navigation message is of the order of 1 m [11], the scale effect will be 0.05 ppm. This paper focuses on the difference between the basic mathematical processing based on adjustment with broadcast and precise ephemerides. The formal accuracy of precise ephemerides of 2 cm does not affect the accuracy of the resulting vector components [11].

2.2. TROPOSPHERIC AND IONOSPHERIC SIGNAL DELAY

The tropospheric delay has two components, wet and dry. The wet component is difficult to model, but fortunately, it represents only a 10% delay. The dry component, responsible for the rest of the delay, can be modelled more easily. The tropospheric delay is frequency independent so, unlike the ionospheric delay, it cannot be removed by combining measurements from the L1 and L2 GPS signals. Depending on the satellite altitude, the tropospheric delay is about 2.5 m to 25 m from the range measurement [5].

The tropospheric signal delay in the zenith direction is eliminated by corrections calculated according to some of the many tropospheric models. For the needs of the basic mathematical processing of GNSS measurements in the network of permanent stations, the most frequently used HOPFIELD model was chosen. During GNSS measurements, the values of atmospheric parameters (temperature, atmospheric pressure, and partial pressure of water vapour) were not measured but calculated from the standard model of the atmosphere.

The ionosphere acts as a dispersive medium, which means that the ionospheric delay depends on the frequency. This delay represents one of the significant errors in the GNSS positioning range and, in some situations, can reach a value of 300 ns (100 m) [12]. The ionospheric delay of the first-order signal is fully captured by the formation of linear frequency combinations IONO FREE from the performed two-frequency GNSS measurements [13]. Second-order ionospheric delay is not treated because it is completely negligible for vector lengths in the GPS permanent stations network.

2.3. EUROPEAN TERRESTRIAL REFERENCE SYSTEM 89

The choice of precise ephemerides for the basic mathematical processing also raised the question of the reference system. Precise satellite ephemerides refer to the ITRS reference system in the measurement epoch, while the SRPOS permanent station coordinates refer to the ETRS89 reference system. The two systems diverge from each other by definition because the ETRS89 reference system is connected to the Eurasian lithosphere plate and moves together with it at a speed of about 2.5 cm/year. In the past 30 years, this distance has reached 70 cm, corresponding to a scale effect of 0.03 ppm, or 1.5 mm in the length of the GITE - GIBL vector. Due to the error below the level of measurement noise during the basic mathematical processing, the transformation of precise ephemerides from the reference system ITRS into the reference system ETRS89 was not performed.

In 1991, the IAG Subcommittee on the European Reference Framework EUREF recommended that the state reference system for Europe coincides with the ITRS in the epoch t_0 = 1989.0 and is connected with the stable part of the Eurasian plate. It is called the European Terrestrial Reference System 89 (ETRS89). Currently, ETRS89 is a regional European system derived from ITRS and is used as a coordinate system throughout Europe [14].

The practical implementation of ETRS89 was initially provided by 93 stations covering Western European countries (European Economic Community, Scandinavian countries, and Austria and Switzerland). Base stations provided laser observations towards satellite targets (SLR / LLR) as well as VLBI quasar readings. A GPS campaign was launched in May 1989, and GPS observations were also used to establish EUREF [15]. Since 1997, the ETRS89 system has been applied by the permanent EUREF network.

3. THE REPUBLIC OF SRPSKA POSITIONING SYSTEM – SRPOS NETWORK

Implementing a network of permanent GNSS stations is a precondition for accurate recording and setting of boundaries, which provides new opportunities in establishing an up-to-date register of landowners, as well as eliminating errors in land registers and cadastral records. The service was supposed to provide GNSS measurements for positioning on the entire Republic of Srpska territory in real time with different accuracy levels by applying one of its RTK and DGPS operating modes and PP mode for subsequent data processing.

The service of permanent GNSS stations of the Republic of Srpska SRPOS was formally launched on September 27, 2011. The Network of Permanent Stations of the Republic of Srpska (SRPOS) currently includes 44 GPS permanent stations, 23 on the territory of the Republic of Srpska, 12 on the territory of the Federation of BiH, four on the territory of Serbia, two on the territory of Croatia and three on the territory of Montenegro.

ETRF2000 was chosen for the epoch 2011.307 as the final reference frame in which the coordinates of the permanent stations of the SRPOS network were calculated. These coordinates represent the position of permanent stations in the ETRS89 reference system.

3.1. INCLUSION OF NEW STATIONS IN THE SRPOS NETWORK

Locations for the installation of new GIBL, GIDE, and GITE permanent stations were selected under the following conditions:

- the location of the permanent station provides a space without physical obstacles above the vertical boundary angle of 10˚,
- the location of the permanent station provides power supply and has other necessary infrastructure for broadcasting corrections,
- the location of the permanent station is protected from lightning strikes,
- there are no sources of strong radio radiation near the location of the permanent station (high voltage lines, transformer stations, radar systems, etc.).

Due to the need to determine the exact coordinates of the permanent stations, the choice of locations for their placement was defined by the relative proximity of the permanent stations of the SRPOS network. The GIBL permanent station is located 2.6 km northwest of the SRPOS permanent station Banja Luka, the GIDE permanent station is located 0.2 km west of the SRPOS permanent station Derventa, and the GITE permanent station is located 0.3 km east of the SRPOS permanent station Teslić (Figure 2).

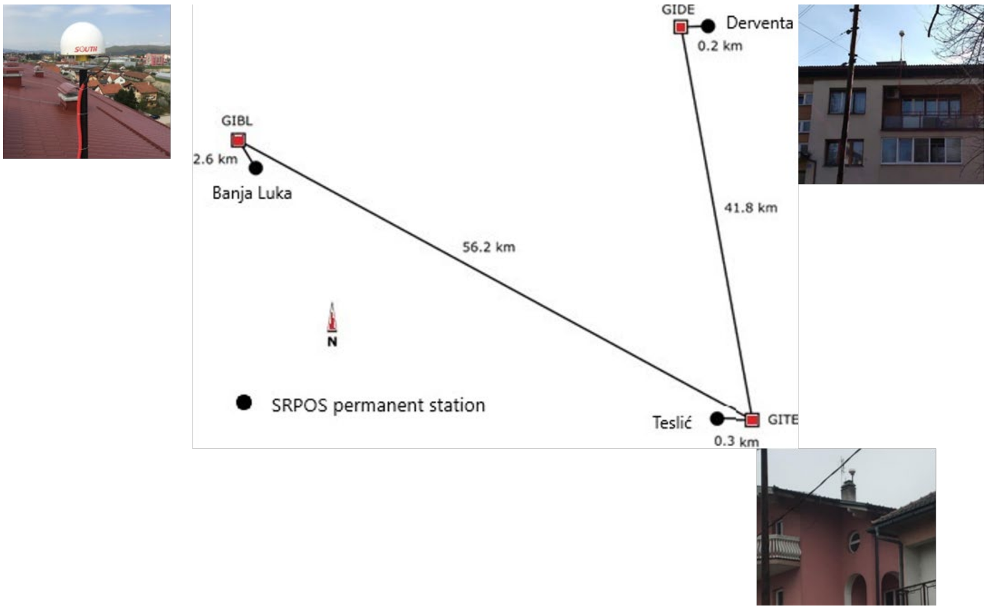

Figure 2. *Representation of GNSS vectors selected for basic mathematical processing and the immediate location of the GIBL, GIDE, and GITE permanent stations*

3.2. MEASUREMENTS IN THE NETWORK OF PERMANENT STATIONS

In the network of permanent stations, NetS8 + and NetS9T GNSS receivers and CHOKE RING GNSS antennas manufactured by SOUTH SURVEYING & MAPPING TECHNOLOGY CO., LTD were used. The NetS8 + and NetS9T GNSS receivers are specifically designed to work on permanent stations. CHOKE RING antennas are widely represented on permanently operating stations due to the excellent performance of multiple reflection filtering and the accuracy of the position of phase centres.

Measurements in the network were performed for five days, from August 8, 2020, to August 12, 2020. The length of the measured session was 24 hours, while the vertical boundary angle was 10˚, and the data registration interval was 15 seconds.

3.3. DOWNLOADED DATA

Daily observation files for SRPOS permanent stations Banja Luka, Derventa, and Teslić, in the industrial standard format RINEX, were taken from the Republic Administration for Geodetic and Property-Legal Affairs in Banja Luka and are shown below (Table 2), as well as the coordinates for these permanent stations (Table 3).

***Table 2**. Downloaded daily RINEX observation files*

Measuring day	BANJA LUKA	DERVENTA	TESLIĆ
08/08/2020	balu221w00.rnx	derv221w00.rnx	tesl221w00.rnx
09/08/2020	balu222w00.rnx	derv222w00.rnx	tesl222w00.rnx
10/08/2020	balu223w00.rnx	derv223w00.rnx	tesl223w00.rnx
11/08/2020	balu224w00.rnx	derv224w00.rnx	tesl224w00.rnx
12/08/2020	balu225w00.rnx	derv225w00.rnx	tesl225w00.rnx

***Table 3**. List of coordinates of permanent stations*

Station	X [m] Y [m] Z [m]	φ [dms] λ [dms] h [m]
Banja Luka	4332148.9575	44° 46' 23.53323"
	1342950.2204	17° 13' 23.67717"
	4469642.6240	214.6830
Derventa	4300617.8700	44° 58' 36.64918"
	1389651.9080	17° 54' 25.63116"
	4485681.2370	215.8995
Teslić	4329889.2046	44° 58' 36.64918"
	1394955.9788	17° 51' 26.38906"
	4456055.2474	270.9276

Navigation files with precise satellite ephemerides in a standard SP3 format (Table 4) were downloaded from the official website of the International GNSS Service (IGS) for a total of 9 days (5 measurement days and two days before and after the measurement campaign).

***Table 4.** Downloaded daily SP3 navigation files (https://www.igs.org/)*

Date	Precise ephemerides
06/08/2020	igs21174.sp3
07/08/2020/	igs21175.sp3
08/08/2020/	igs21176.sp3
09/08/2020	igs21181.sp3
10/08/2020/	igs21182.sp3
11/08/2020/	igs21183.sp3
12/08/2020/	igs21184.sp3
13/08/2020/	igs21185.sp3
14/08/2020/	igs21186.sp3

4. RESULTS

In the first step of this paper, the downloaded RINEX data were imported, and all data were checked. In the first case, broadcast ephemerides were used, and in the second case, published precise ephemerides were imported. Based on a predefined observation plan, baselines (vectors) were processed. During the processing, it was checked whether the phase uncertainties were solved for all vectors, that is, whether fixed solutions were obtained for all vectors. Published precise ephemerides were included to obtain the highest quality and most accurate adjustment results.

Using the least-squares method, the results of aligned base vectors were obtained, i.e. their aligned components as well as variance and covariance. The adjustment results represent the most probable values of point coordinates.

4.1. MATHEMATICAL PROCESSING OF GPS VECTOR

After entering the points and checking all the entered data, baselines (vectors) are processed. In data processing in order to obtain the components of the base vector between two points, mathematical models of relative positioning are used. The processing of baselines is done based on the observation plane and network sketch, so it is predetermined which baselines will enter the adjustment of the network. Manual processing of baselines was used when adjusting the subject network.

Simultaneous GNSS measurements on six permanent stations allow the formation of a total of five independent GNSS vectors. Such a choice is not unequivocal. The adopted approach coincides with the approach applied in EUREF thickening measurement campaigns and consists of a selection of independent vectors that meet two criteria. The first criterion is that the vectors are interconnected, and the second is that they collectively have the smallest length.

Following the adopted approach, the basic mathematical processing of the performed GNSS measurements was performed for the vectors: Banja Luka - GIBL, Derventa - GIDE, Teslić - GITE, GITE - GIBL, and GITE - GIDE. The total length of the selected vectors is 101.1 km (Figure 2).

4.2. CALCULATION OF DAILY SOLUTIONS

The coordinates of permanent stations were determined individually for each day within the adjustment by the method of least squares. The daily adjustment of the permanent station network was performed using a mathematical model that included the following wholes: the functional model, stochastic model, and date defect.

The functional model of adjustment consists of measured and unknown quantities. The measured quantities are components of the GNSS vector obtained within the basic mathematical processing of the initial measurement results, and the unknown quantities are the three-dimensional coordinates of permanent stations. The stochastic model is represented by a quasi-diagonal matrix whose diagonal members of the covariance matrix are represented by the covariences of the individual vector components and are obtained in the basic mathematical processing of the initial measurement results. The date defect is three, and it is eliminated in the adjustment process by fixing the SRPOS coordinates of the permanent station Banja Luka.

The adjustment parameters that were common to each daily solution are presented in Table 5.

***Table 5**. Common parameters of daily adjustments*

Parameter	Value
Total number of vectors	5
Total number of measurements	15
Total number of points	6
Total number of unknowns	18
Date defect	3
Significance level of one-dimensional test	5.0 %
Significance level of multidimensional test	1.0 %
The power of the test	80 %
A priori standard unit of weight	10

The quality of the coordinates of permanent stations obtained by daily solutions was assessed from the point of view of their consistency from day to day. Quality indicators are determined in two ways. The first method involved the calculation of standard deviations of daily solutions from their deviations from the arithmetic mean, and the second method involved the calculation of a unified estimate of standard deviation from standard deviations of daily solutions.

The standard deviations of the horizontal and vertical positions of permanent stations along the axes of the geocentric coordinate system, when broadcast ephemerides were used, are respectively:

$\sigma_{HP} = 428.8 \text{ mm}$

$\sigma_{VP} = 104.0 \text{ mm}.$

Standard deviations of the position of permanent stations, when precise ephemerides are included in the adjustments, are:

$\sigma_{HP} = 2.9 \text{ mm}$

$\sigma_{VP} = 4.9 \text{ mm}.$

The standard deviations of the horizontal and vertical positions of the permanent station along the axes of the local geodetic system are:

- With broadcast ephemerides:

$\sigma_{HP} = 71.1\ mm$

$\sigma_{VP} = 54.2\ mm$

- With precise ephemerides:

$\sigma_{HP} = 2.6\ mm$

$\sigma_{VP} = 4.4\ mm$

The equality of dispersions of horizontal and vertical positions obtained in the two presented ways was tested by Fisher's test with a significance level of 5% and a confidence level of 95%, respectively:

- With broadcast ephemerides:

$$F_{HP} = \frac{428.8^2}{71.1^2} = 36.37 > 2.37 = F_{(0.95,4,\infty)}$$

$$F_{VP} = \frac{104.0^2}{54.2^2} = 3.68 > 2.37 = F_{(0.95,4,\infty)}$$

- With precise ephemerides:

$$F_{HP} = \frac{2.9^2}{2.6^2} = 1.24 < 2.37 = F_{(0.95,4,\infty)}$$

$$F_{VP} = \frac{4.9^2}{4.4^2} = 1.24 < 2.37 = F_{(0.95,4,\infty)}$$

When we look at the testing of the equality of dispersions of horizontal and vertical positions obtained in two ways, it can be concluded that the coordinate accuracy is not consistent with the daily variations of the coordinates of permanent cells when broadcast ephemerides were used in the adjustment.

According to the results of Fisher's test, the estimates of standard deviations obtained from the deviation from the arithmetic mean fully correspond to the estimates of standard deviations of daily solutions obtained by adjustment if precise ephemerides are included in the adjustment. It can be concluded that the accuracy of the coordinates is consistent with daily variations of the coordinates of the permanent stations.

4.3. CALCULATION OF THE DEFINITIVE SOLUTION

To control the concurrence of the coordinates of the network of permanent stations with the official SRPOS coordinates of the permanent stations Banja Luka, Derventa, and Teslić, the network of permanent stations was adjusted as free by the method of least squares.

The date defect of the network was eliminated in the adjustment process by fixing the values of SRPOS coordinates of the permanent station Banja Luka.

During the adjustment of the subject network, before the processing of the vector between the measured points, the merging of the measurement intervals (their joining) was performed for each point individually. Free adjustment of the network was performed with the parameters shown in Table 6.

Table 6. *Free adjustment parameters*

Parameter	Value
Total number of vectors	25
Total number of measurements	75
Total number of points	6
Total number of unknowns	18
Date defect	3
Significance level of one-dimensional test	5.0 %
Significance level of multidimensional test	0.5 %
The power of the test	80 %
A priori standard unit of weight	10

A sketch of the grid after adjustment, with error ellipses, is shown in Figure 3 and Figure 4.

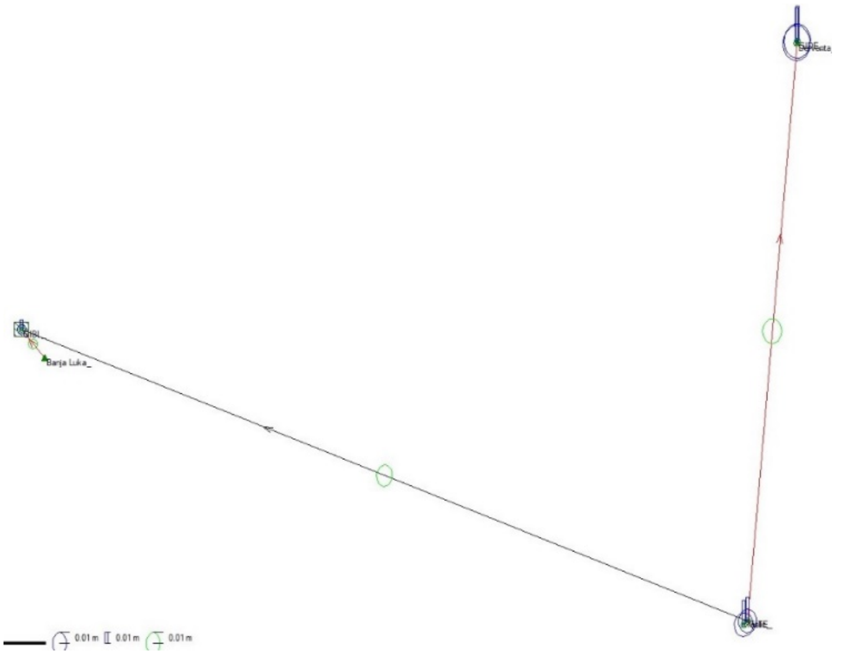

Figure 3. *Sketch of the grid after the adjustment with broadcast ephemerides*

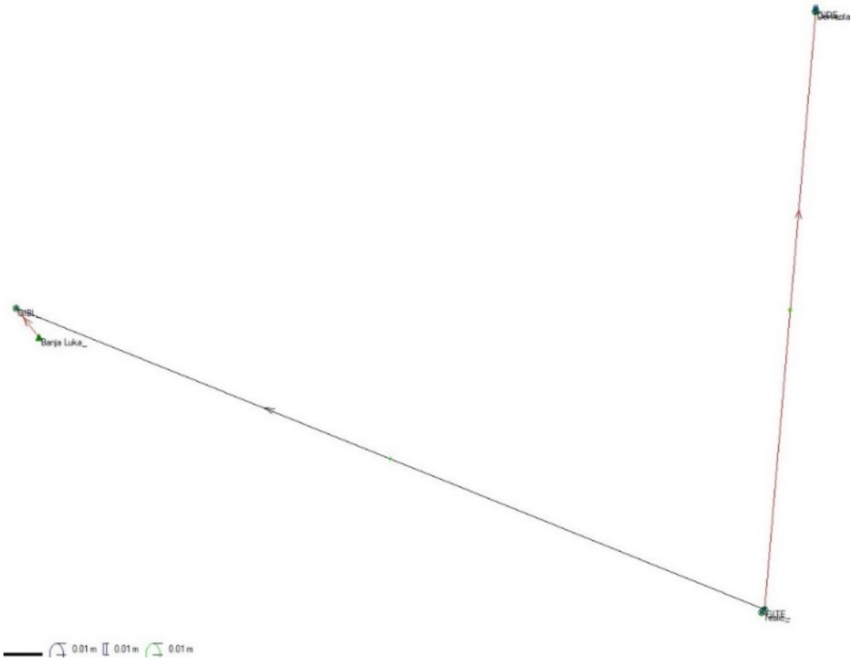

Figure 4. *Sketch of the grid after the adjustment with precise ephemerides*

Table 7 shows the coordinates for permanent stations after the free adjustment in the WGS84 and ETRS89 coordinate system when broadcast and precise ephemerides were used. Table 8 shows standard deviations by coordinate axes and standard position deviations.

Table 7. *Coordinates of stations after free adjustment*

With broadcast ephemerides			With precise ephemerides		
Station	X, Y, Z [m]	E, N, H [m]	Station	X, Y, Z [m]	E, N, H [m]
Banja_Luka	4332148.9575	201159.9311	Banja_Luka	4332148.9575	201159.9311
	1342950.2205	4964698.6641		1342950.2205	4964698.6641
	4469642.6241	214.6830		4469642.6241	214.6830
Derventa	4300617.8555	256135.3335	Derventa	4300617.8599	256135.3361
	1389651.9066	4985032.9840		1389651.9110	4985032.9800
	4485681.2197	215.8773		4485681.2197	215.8811
Teslić	4329889.2047	250593.4652	Teslić	4329889.2064	250593.4679
	1394955.9755	4943403.3392		1394955.9791	4943403.3354
	4456055.2397	270.9216		4456055.2372	270.9218
GIBL	4331278.3938	199615.8870	GIBL	4331278.3978	199615.8848
	1340964.7502	4966795.2555		1340964.7494	4966795.2500
	4471081.6382	221.0718		4471081.6340	221.0714
GITE	4329640.8058	250829.0047	GITE	4329640.8076	250829.0072
	1395112.9350	4943655.7281		1395112.9384	4943655.7246
	4456236.3784	264.0464		4456236.3764	264.0470
GIDE	4300517.3739	256172.4910	GIDE	4300517.3778	256172.4928
	1389653.6563	4985152.4731		1389653.6596	4985152.4699
	4485756.9660	202.1616		4485756.9663	202.1652

Table 8. *Standard deviations obtained by free adjustment and standard position estimates*

With broadcast ephemerides				With precise ephemerides			
Station	σ_X [mm] σ_Y [mm] σ_Z [mm]	σ_E [mm] σ_N [mm] σ_H [mm]	σ_{HP} [mm]	Station	σ_X [mm] σ_Y [mm] σ_Z [mm]	σ_E [mm] σ_N [mm] σ_H [mm]	σ_{HP} [mm]
Banja_Luka	0.0	0.0	0.0	Banja_Luka	0.0	0.0	0.0
	0.0	0.0			0.0	0.0	
	0.0	0.0			0.0	0.0	
Derventa	25.3	12.1	19.9	Derventa	3.3	1.6	2.6
	13.9	15.7			1.8	2.1	
	24.5	32.2			3.2	4.2	
GIBL	6.7	4.0	5.9	Teslić	0.7	0.3	0.5
	4.3	4.4			0.4	0.4	
	7.0	8.8			0.7	0.9	
GIDE	24.6	11.8	19.3	GIBL	3.2	1.5	2.5
	13.5	15.3			1.8	2.0	
	23.8	31.3			3.1	4.1	
GITE	16.6	8.1	13.2	GITE	1.8	0.8	1.4
	9.3	10.4			1.0	1.1	
	16.2	21.2			1.7	2.3	
Teslić	17.1	8.4	13.6	GIDE	1.9	0.9	1.4
	9.5	10.7			1.0	1.2	
	16.6	21.8			1.8	2.3	

5. CONCLUSION

This paper aims to analyse the accuracy of the coordinates of permanent stations using the least-squares method by processing broadcast and precise ephemerides. It is important to note that the figures were not closed during the network formation.

All used permanent stations are located in the area of Banja Luka, Teslić, and Derventa and are located at a spatial distance of approximately 50 km. Static relative positioning is known to be a very precise positioning method that achieves the accuracy of up to 0.1 ppm (0.1 mm / km) in the vertical view and up to 0.4 ppm (0.4 mm / km) in the horizontal view [16]. Due to these baseline lengths, it makes sense to analyse further and quantify the impact of broadcast and precise ephemerides on the newly determined coordinates.

The vertical component accuracy is worse than for the horizontal component, which is the expected situation and fact, which arises from the general design of the GNSS system. In both cases, using broadcast and precise ephemerides, it can be said that the observation time for the three components (north, east, and up) affects the accuracy; as the observation time increases, so does the accuracy. Components such as satellite availability and visibility, ambiguity, and ambiguous GNSS signal paths in short observation times affect the accuracy from time to time.

In the first part of the research, adjustments were conducted by days, in the first case using broadcast and in the second case using precise ephemerides. The value of Fisher's test statistics for 95% confidence is 2.37. When we look at the testing of the equality of dispersions of horizontal and vertical positions obtained in two ways, it can be concluded that the coordinate accuracy is not consistent with the daily variations of coordinates of permanent stations when broadcast ephemerides were used in the adjustment, because their ratio at the horizontal position is 36.37 mm and at the vertical position 3.68 mm, which is greater than the test statistic. The estimates of standard deviations obtained from the deviation from the arithmetic mean fully correspond to the estimates of standard deviations of daily solutions obtained by the adjustment if precise ephemerides are included in the adjustment because their ratio in both horizontal and vertical positions is 1.24 mm. This value is less than the test statistic value of 2.37 mm, so it can be concluded that the coordinate accuracy is consistent with the daily variations of the coordinates of the permanent stations.

We obtained the coordinates of the newly determined permanent stations in the WGS84 system and the ETRS89 system by free adjustment. Standard deviations of the horizontal and vertical position showed much larger deviations when broadcast ephemerides were used in the adjustment. The largest deviation was $\sigma_{HP} = 19.9\ \mathrm{mm}$ and $\sigma_{VP} = 28.35\ \mathrm{mm}$for the permanent station Derventa. When precise ephemerides for the same station were used in the adjustment deviations were $\sigma_{HP} = 2.6\ \mathrm{mm}$ and $\sigma_{VP} = 3.7\ \mathrm{mm}$. The smallest standard deviations of the horizontal and vertical position were obtained for the GIBL permanent station, with broadcast ephemerides $\sigma_{HP} = 5.9\ \mathrm{mm}$ and $\sigma_{VP} = 7.9\ \mathrm{mm}$, and with precise ephemerides $\sigma_{HP} = 0.5\ \mathrm{mm}$ and $\sigma_{VP} = 0.8\ \mathrm{mm}$.

Finally, it can be concluded that the use of precise ephemerides impacts the accuracy of determining the coordinates of newly included stations for baseline lengths exceeding 50 km. Therefore, in practice, when adjustment networks whose span between points exceeds 50 km, the adjustment procedure should be carried out using the adopted precise ephemerides for a certain observation period. For baseline lengths of 30 to 50 km, precise ephemerides are recommended, and for shorter lengths, adjustment with broadcast ephemerides achieves a satisfactory accuracy.

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