University of Banja Luka I Faculty of Architecture, Civil Engineering and Geodesy
ISSN 2712-057 I E-ISSN 2303-6036 I UDC 72

AGG+

2022_10(1)

AGG+ Journal for Architecture, Civil Engineering, Geodesy and Related Scientific Fields
АГГ+ часопис за архитектуру, грађевинарство, геодезију и сродне научне области

002-015

Categorisation I Original scientific paper
DOI I 10.7251/AGGPLUS/2210048J
UDC I 539.434.014:669.721
COBISS.RS-ID I XXXXXXXXXXXXXXXXX
Paper received I 13/08/2022
Paper accepted I 04/09/2022

Miloš Jočković
University of Belgrade, Faculty of Civil Engineering, Serbia, mjockovic@grf.bg.ac.rs, https://orcid.org/0000-0002-2409-9450

Marija Nefovska-Danilović
University of Belgrade, Faculty of Civil Engineering, Serbia, marija@grf.bg.ac.rs, https://orcid.org/0000-0002-2613-1952

Aleksandar Borković
Graz University of Technology, Institute for Applied Mechanics, Austria, aborkovic@tugraz.at, https://orcid.org/0000-0002-4091-3379

LINEAR DYNAMIC ANALYSIS OF A SPATIALLY CURVED BERNOULLI-EULER BEAM SUBJECTED TO MOVING LOAD

Original scientific paper
DOI 10.7251/AGGPLUS/2210048J
UDC 539.434.014:669.721

COBISS.RS-ID XXXXXXXXXXXXXXXX
Paper received I 13/08/2022
Paper accepted I 04/09/2022

* corresponding author

This paper is an extended version of the paper previously published in the Proceedings of STEPGRAD2022 International Conference.

Miloš Jočković *

University of Belgrade, Faculty of Civil Engineering, Serbia, mjockovic@grf.bg.ac.rs, https://orcid.org/0000-0002-2409-9450

Marija Nefovska-Danilović

University of Belgrade, Faculty of Civil Engineering, Serbia, marija@grf.bg.ac.rs, https://orcid.org/0000-0002-2613-1952

Aleksandar Borković

Graz University of Technology, Institute for Applied Mechanics, Austria, aborkovic@tugraz.at, https://orcid.org/0000-0002-4091-3379

LINEAR DYNAMIC ANALYSIS OF A SPATIALLY CURVED BERNOULLI-EULER BEAM SUBJECTED TO A MOVING LOAD

ABSTRACT

This paper considers the dynamic analysis of a spatially curved Bernoulli-Euler beam subjected to a moving load. The isogeometric approach is used for the spatial discretization of the weak form of the equation of motion. Both the reference geometry and the solution space are represented using the same NURBS basis functions that guarantee an accurate description of the beam centerline. The time integration is done by the explicit technique. The presented formulation is validated by the comparison with the existing results from the literature for the curved beam subjected to a constant load moving with a constant velocity. In addition, the influence of the moving load velocity on the dynamic response of a spatially curved beam has been investigated.

Keywords: *isogeometric approach, Bernoulli-Euler curved beam, moving load*

1. INTRODUCTION

Beam-like structures are often subjected to dynamic loads during their lifetime. Consequently, numerical methods are essential for more accurate and reliable prediction of their dynamic behavior. The standard dynamic load case for cranes and bridges is a mass that moves along the structure. The moving mass is usually modeled as a moving force with constant magnitude and direction. Such an approach is referred to as a "moving load" model, where the inertial term of the moving mass is neglected. The majority of the research in this field is related to the analysis of a mass moving along a straight beam. One of the earliest investigations was carried out by Stokes in 1849 [1], where the influence of the moving mass on the plane straight Bernoulli-Euler beam was considered analytically using the moving load model.

Due to the aesthetic and functional requirements in the design process, curved spatial beam elements are common in structural engineering. The geometrical model of the curved spatial beam requires the spatial curve, which is usually defined using computer-aided design (CAD) software packages. To accurately describe the free-form curves and the curves of conic sections such as a circle, ellipse, parabola and hyperbola, CAD packages utilize the NURBS (Non-Uniform Rational B-Spline) basis functions. Furthermore, the computation of the dynamic response of complex spatially curved beams in practical applications is performed using the Finite element method (FEM), which is implemented in many software packages for structural analysis. A direct relation between the CAD and FEM has not yet been established [2], leading to a costly and time-consuming iterative design process. The isogeometric approach establishes a direct relationship between the geometry and the unknown fields of the structure [2]. This is enabled by using the NURBS functions as basis functions for both reference geometry and solution spaces of a numerical model. Therefore, the same basis functions are applied for the geometry and kinematics, which eliminates the errors due to the geometric approximation in a spatially discretized model. In order to improve the mesh, three types of mesh refinement are used in the isogeometric approach, denoted as H-, P-, and K-methods [2].

A dynamic analysis of an arbitrarily curved spatial beam subjected to a moving load is studied in this paper. A short review on the NURBS basis functions is given in Section 2 and followed by a representation of the beam geometry. The governing equation of motion of the Bernoulli-Euler isogeometric beam element is briefly given in Section 4, while more details can be found in the authors' previous paper [3]. The moving load model is presented in Section 5, followed by the numerical example of a spatially curved beam subjected to the moving load presented in Section 6. At the end, the main conclusions have been drawn.

2. BASICS OF NURBS

The exact shape of an arbitrary curve $\mathbf{C}(\xi)$ in Euclidean 3D space can be represented as:

$$\mathbf{C}(\xi) = \sum_{i=1}^{n} R_{i,p}(\xi)\mathbf{C}_i \tag{1}$$

where $R_{i,p}(\xi)$ is the i-th NURBS basis function, p is the function degree, $\mathbf{C}_i$ is the position of the control point i, while n is the number of basis functions and control points. NURBS functions are derived from the B-spline functions:

$$R_{i,p}(\xi) = \sum_{i=1}^{n} \frac{N_{i,p}(\xi)\cdot w_i}{\sum_{j=1}^{n} N_{j,p}(\xi)\cdot w_i} \tag{2}$$

where w_i is the i-th function weight. In order to define B-spline functions, the Cox de Boor algorithm is often applied [4].

For the case of a zero degree (p = 0), the B-spline functions are defined as:

$$N_{i,0}(\xi) = \begin{cases} 1, & \text{if } \xi \in [\xi_i, \xi_{i+1}[\\ 0, & \text{otherwise} \end{cases} \tag{3}$$

while for the polynomial degree greater than zero (p > 0):

$$N_{i,p}(\xi) = \begin{cases} \dfrac{\xi - \xi_i}{\xi_{i+p} - \xi_i} N_{i,p-1}(\xi) + \dfrac{\xi_{i+p-1} - \xi}{\xi_{i+p+1} - \xi_{i+1}} N_{i+1,p-1}(\xi), & \text{if } \xi \in [\xi_i, \xi_{i+p+1}[\\ 0, & \text{otherwise} \end{cases} \tag{4}$$

The B-spline functions are polynomial functions defined in a parametric domain (ξ) using the knot vector. This vector represents a set of non-decreasing real numbers, the knots.

Important properties of the B-spline function, as well as the NURBS functions, used in the following derivations, are the non-negativity and the partition of unity over the parametric domain. More about the B-spline and NURBS basis functions can be found in [4].

As mentioned in the previous section, there are several important features of the NURBS-based parameterization. For example, it is possible to exactly describe the initial smooth geometries, which promises more accurate simulations. Furthermore, besides standard H- and P- refinement strategies, the isogeometric approach allows the definition of an interelement continuity up to C^{p-1}, known as K-refinement. The high smoothness of the kinematic field often returns improved convergence rates [5,6].

3. BEAM GEOMETRY

Due to the assumptions of beam theories, all kinematic and stress quantities of a beam can be given as a function of the beam centerline. In general, the beam centerline has an arbitrary shape in the Euclidean three-dimensional space, forming a curved line. The formulation of a curved beam is conducted using the curvilinear coordinate system attached to the beam centerline.

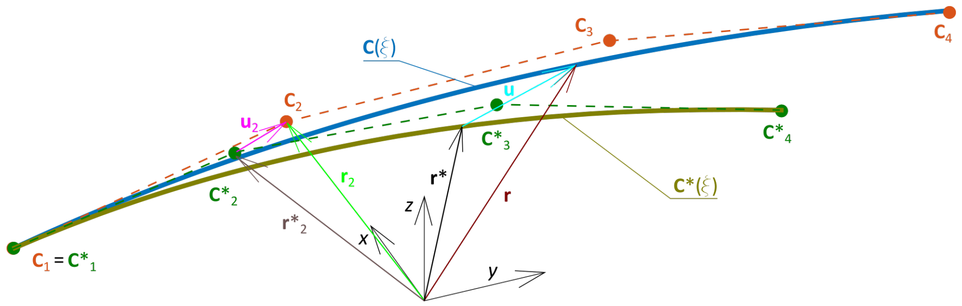

Figure 1. Centerline of a curved beam with corresponding control points

Using the NURBS parameterization, the position vector of a curved line is defined as:

$$\mathbf{r}(\xi)=\sum_{i=1}^{n}R_{i,p}(\xi)\mathbf{r}_i \tag{5}$$

where $\mathbf{r}_i$ is the position of the *i*-th control point, Figure 1. To fully define the beam continuum, a unique triad must be attached to each point of a curve. Here, this triad is aligned with the Frenet-Serret frame. The basis vectors are defined using the well-known relations of differential geometry [7] and the relations between the arc-length and NURBS parameterizations:

$$\mathbf{g}_1=\mathbf{r}_{,1}=\frac{d\mathbf{r}}{d\xi}=\frac{d\mathbf{r}}{ds}\frac{ds}{d\xi}=\mathbf{t}\frac{ds}{d\xi}=\mathbf{t}\sqrt{g_{11}}$$

$$\mathbf{g}_2=\mathbf{n}=\frac{1}{K}\frac{d\xi}{ds}\frac{d}{d\xi}\left(\frac{\mathbf{g}_1}{|\mathbf{g}_1|}\right) \tag{6}$$

$$\mathbf{g}_3=\mathbf{b}=\frac{\mathbf{g}_1\times\mathbf{n}}{|\mathbf{g}_1\times\mathbf{n}|}$$

where **t**, **n** and **b** are orthonormal basis vectors of the beam centerline obtained using arc-length parameterization (Frenet-Serret frame of reference), while $\mathbf{g}_1$, $\mathbf{g}_2$ and $\mathbf{g}_3$ form orthogonal vector basis with respect to the parametric coordinate. The vector $\mathbf{g}_1$ is collinear with the tangent **t**, while the vectors $\mathbf{g}_2$ and $\mathbf{g}_3$ are in the beam cross-section plane. In the previous relations, K is the modulus of curvature, while g_{11} is the component of the metric tensor of the beam centerline:

$$g_{ij}=\begin{bmatrix} g_{11} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},\ \det\left(g_{ij}\right)=g_{11}=g \tag{7}$$

By using the well-known Frenet-Serret relations and Eq. (6), the derivatives of the basis vectors with respect to the parametric coordinate are:

$$\begin{bmatrix} \mathbf{g}_{1,1} \\ \mathbf{g}_{2,1} \\ \mathbf{g}_{3,1} \end{bmatrix} = \begin{bmatrix} \Gamma^1_{11} & gK & 0 \\ -gK & 0 & \sqrt{g}\tau \\ 0 & -\sqrt{g}\tau & 0 \end{bmatrix} \begin{bmatrix} \mathbf{g}_1 \\ \mathbf{g}_2 \\ \mathbf{g}_3 \end{bmatrix} \tag{8}$$

where $()_{,1}$ represents the derivative with respect to the parameter ξ, Γ^1_{11} is the Christoffel symbol of the second kind, and τ is the torsion of the beam centerline.

In this paper, the beam cross-section principal axes coincide with the basis vectors $\mathbf{g}_2$ and $\mathbf{g}_3$. If this condition is not satisfied, the basis vectors $\mathbf{g}_2$ and $\mathbf{g}_3$ need to be rotated around the basis vector $\mathbf{g}_1$ to align them with the principal axes, forming a new moving frame of reference [8].

Using the introduced basis vectors, the position vector of an arbitrary point of the beam can be defined as:

$$\hat{\mathbf{r}} = \mathbf{r} + \eta \mathbf{g}_2 + \zeta \mathbf{g}_3 \tag{9}$$

where η and ζ are the coordinates along the principal axes of the cross-section. Consequently, the first basis vector of an arbitrary point is defined as:

$$\hat{\mathbf{g}}_1 = \frac{d\hat{\mathbf{r}}}{d\xi} = \mathbf{g}_{1,1} + \eta \mathbf{g}_{2,1} + \zeta \mathbf{g}_{3,1} = g_0 \mathbf{g}_1 + \eta K_1 \mathbf{g}_2 + \zeta K_1 \mathbf{g}_3 \tag{10}$$

Due to the assumption of the rigid cross-section, the vectors $\mathbf{g}_2$ and $\mathbf{g}_3$ are translated from the beam centerline to an arbitrary point. From the last equation, it is evident that the basis vector $\hat{\mathbf{g}}_1$ is not perpendicular to the vectors $\mathbf{g}_2$ and $\mathbf{g}_3$. However, in the frame of linear analysis, it is possible to orthogonalize these vectors by introducing a new coordinate system [8,9].

4. ISOGEOMETRIC BERNOULLI-EULER BEAM FORMULATION

Due to the external impact, the beam centerline has a new position defined with the current position vector:

$$\mathbf{r}^* = \mathbf{r} + \mathbf{u} \tag{11}$$

where $\mathbf{u}$ represents the displacement vector of the beam centerline. Using the isogeometric approach, the displacement vector can be represented as:

$$\mathbf{u}(\xi) = \sum_{i=1}^{n} R_{i,p}(\xi)\mathbf{u}_i = \sum_{i=1}^{n} R_{i,p}(\xi) u_i^m \mathbf{i}_m \tag{12}$$

where $\mathbf{u}_i$ is the displacement vector of the i-th control point. Note that the displacement vector and the reference geometry of the beam centerline are represented using the same basis functions, which is the fundamental property of the isogeometric approach.

Formulation of the spatial Bernoulli-Euler isogeometric beam is conducted by applying the convective coordinate system, and the position vector of an arbitrary point of a deformed beam is:

$$\hat{\mathbf{r}}^{*} = \hat{\mathbf{r}} + \eta \mathbf{g}_2^{*} + \zeta \mathbf{g}_3^{*} \tag{13}$$

The basis vectors of the deformed configuration can be expressed as:

$$\mathbf{g}_m^{*} = \mathbf{g}_m + \mathbf{u}_m \tag{14}$$

where $\mathbf{u}_m$ is the gradient of displacement along the m^{th} axis of the (ξ,η,ζ) coordinate system.

Using Eqs. (9), (13) and (14), the displacement vector of an arbitrary point of a beam is defined as:

$$\hat{\mathbf{u}} = \mathbf{u} + \eta \mathbf{u}_2 + \zeta \mathbf{u}_3 \tag{15}$$

Using Eq. (15), the acceleration vector of an arbitrary point is obtained as the second material time derivative:

$$\hat{\mathbf{a}} = \ddot{\left(\hat{\mathbf{u}}\right)} = \ddot{\mathbf{u}} + \eta \ddot{\mathbf{u}}_2 + \zeta \ddot{\mathbf{u}}_3 \tag{16}$$

In addition, the variation of displacement of an arbitrary point is obtained from Eq. (15) as:

$$\delta \hat{\mathbf{u}} = \delta \mathbf{u} + \eta \delta \mathbf{u}_2 + \zeta \delta \mathbf{u}_3 \tag{17}$$

The components of the Green-Lagrange strain tensor are:

$$\hat{\varepsilon}_{ij} = \frac{1}{2}\left(\hat{\mathbf{g}}_i^{*} \cdot \hat{\mathbf{g}}_j^{*} - \hat{\mathbf{g}}_i \cdot \hat{\mathbf{g}}_j\right) = \frac{1}{2}\left(\hat{g}_{ij}^{*} - \hat{g}_{ij}\right) \tag{18}$$

The assumption of rigid cross-cross section returns only three non-zero components of the strain tensor:

$$\begin{aligned} \hat{\varepsilon}_{11} &= \frac{1}{2}\left(\hat{g}_{11}^{*} - \hat{g}_{11}\right) \\ \hat{\varepsilon}_{12} &= \frac{1}{2}\left(\hat{g}_{12}^{*} - \hat{g}_{12}\right) \\ \hat{\varepsilon}_{13} &= \frac{1}{2}\left(\hat{g}_{13}^{*} - \hat{g}_{13}\right) \end{aligned} \tag{19}$$

By substituting the second Bernoulli-Euler assumption of orthogonality between cross-section and centerline into the previous equations, the required kinematic relations are obtained. Degrees of freedom of the isogeometric Bernoulli-Euler beam are the displacements of the beam centerline and the torsional rotation of the beam cross-section. The detail derivations of the kinematic relations can be found in [3].

Assuming the linear elastic material behavior, the constitutive relations can be written as:

$$\hat{S}_i^j = 2\mu\hat{\varepsilon}_i^j + \lambda\delta_i^j\hat{\varepsilon}_m^m \tag{20}$$

where $\hat{S}_j^i$ are the mixed components of the second Piola-Kirchoff stress tensor, while μ and λ are Lame's constants.

In order to obtain the discrete equations of motion, the principle of virtual work is used:

$$\int_{V_0} \rho\hat{\mathbf{a}} \cdot \delta\hat{\mathbf{u}} dV_0 + \int_{V_0} \mathbf{S} : \delta\boldsymbol{\varepsilon} dV_0 = \int_{V_0} \hat{\mathbf{f}} \delta\hat{\mathbf{u}} dV_0 \tag{21}$$

where ρ is the mass density, while $\hat{\mathbf{f}}$ is the external load. By substituting Eqs. (16), (17), (19), and (20) into Eq. (21), the governing equation of the motion of the Bernoulli-Euler isogeometric curved beam subjected to the moving load is obtained:

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{Q} \tag{22}$$

where **M** is the mass matrix, **K** is the stiffness matrix, **Q** is the vector of equivalent control forces, while **q** is the displacement vector of the control points. The solution of this equation requires the application of a time integration procedure. The explicit step-by-step integration has been applied using the finite differences method [10]. The reduced integration has been applied in Eq. (21) [11], and implemented into the original MATLAB code [12].

5. MOVING LOAD

A mass moving along the structure generates a dynamic response. This load can be modeled as a single load with constant magnitude and direction, $\mathbf{f}_0$, that moves along a beam with the constant velocity:

$$\mathbf{f}(t) = \mathbf{f}_0 \cdot \delta\left(\xi - V_\xi t\right)$$

$$V_\xi = \frac{d\xi}{dt} = \frac{ds}{dt}\frac{d\xi}{ds} = \frac{V}{\sqrt{g}} \tag{23}$$

where V_ξ and V are the magnitudes of velocity with respect to the parametric and arc-length coordinates, respectively.

The vector of equivalent forces of the *i*-th control point in the case of a point load is:

$$\mathbf{Q}_i = \int_{d\xi} \mathbf{f} \cdot R_{i,p}\left(\xi\right)\sqrt{g} d\xi = \mathbf{f} \cdot R_{i,p}\left(\xi_m\right)\sqrt{g} \tag{24}$$

where ξ_m is the position of the moving load on a beam.

6. NUMERICAL EXAMPLES

6.1. VALIDATION AND CONVERGENCE STUDY

The validation study of the proposed formulation is given in this section. A horizontally curved arch in the x-y plane with the length L = 24 m and the subtended angle α = 30° is subjected to the out-of-plane and in-plane moving load with constant speed V = 40 m/s. The displacements and the torsional rotations at both ends of the beam are restrained. The beam geometry has been modeled with the cubic NURBS, as given in Figure 2.

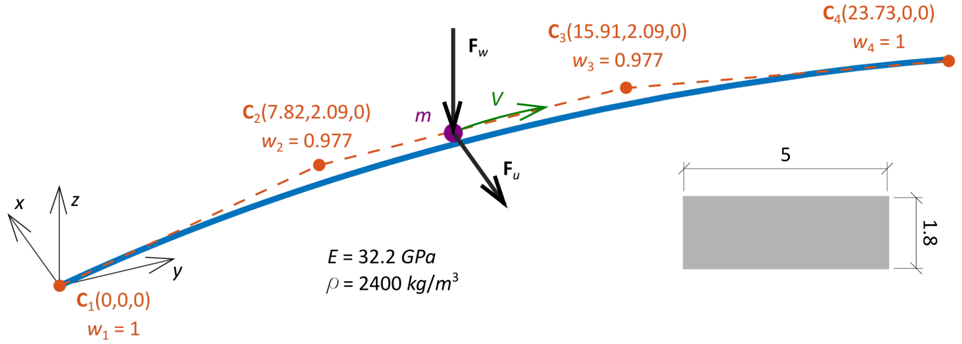

Figure 2. The arch subjected to the moving load

The material is homogeneous and defined using the Young's modulus E = 32.2 GPa, the Poisson's ratio ν = 0.2 and the mass density ρ = 2400 kg/m^3, while the cross-section is rectangular with the dimensions b/h = 5/1.8 m. The beam is subjected to the out-of-plane load $\mathbf{F}_w$ = -293.32 kN and the in-plane load $\mathbf{F}_u$ = 1043.71 kN directed towards the arch center. The displacements of the beam midpoint obtained using the isogeometric approach have been compared with the semi-analytical results from the literature, applicable only for simply supported arches [13]. It is important to point out that the beam model presented in [13] is based on the Timoshenko beam theory. In this example, the validation study, as well as the convergence study, are conducted using the P-refinement procedure.

The in-plane (u) and the out-of-plane (w) displacement components of the midpoint obtained using the P-refinement procedure are presented respectively in Figure 3 and Figure 4.

In addition, the same example is used to calculate the influence line of the beam midpoint displacement components by neglecting the inertial part of the beam in the principle of virtual work.

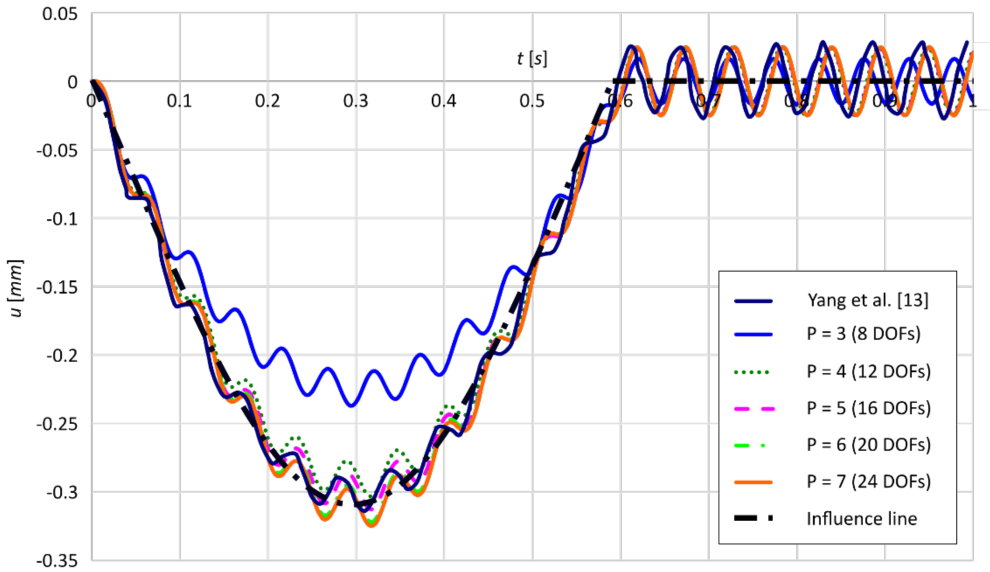

Figure 3. Comparison of the in-plane displacement component (u) of the beam midpoint

By comparing the results of the beam midpoint displacements obtained using the dynamic and static analysis, a significant difference can be observed, especially for the case of the out-of-plane displacement.

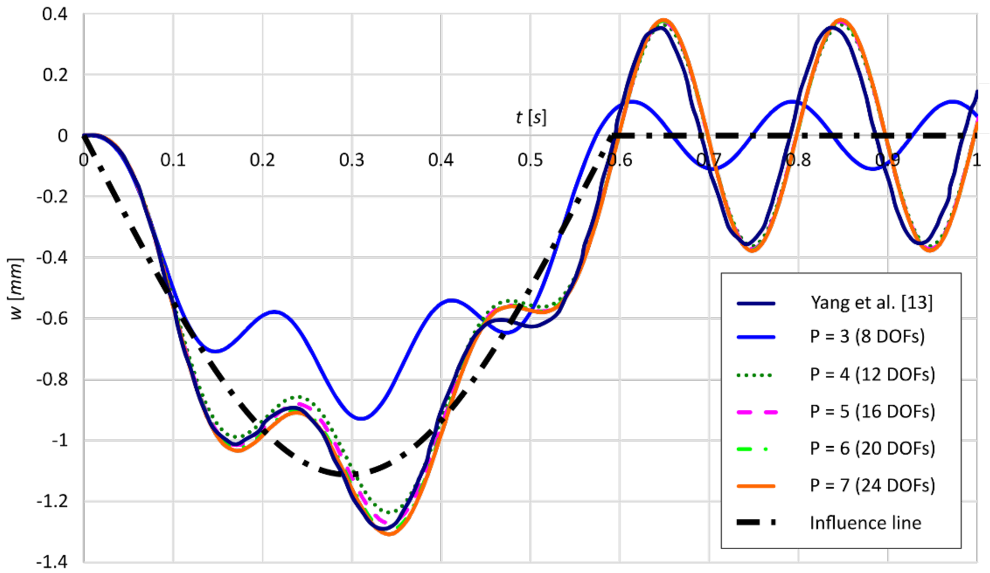

Figure 4. Comparison of the out-of-plane displacement component (w) of the beam midpoint

6.2. PARAMETRIC STUDY

In this example, the effects of the moving load velocity on the dynamic response of a curved cantilever beam are investigated. The geometry of the beam is defined using five control points with a unit weight vector and 3rd order B-Spline basis functions, Figure 5. The beam is clamped at the first beam control point $\mathbf{C}_1(0,0,0)$. The beam material is defined using the Young's modulus E = 32.2 GPa, the Poisson's ration ν = 0.2 and the mass density ρ = 2400 *kg*/*m*^{3}. The cross-section of the beam is circular with the diameter *R* = 0.1 *m*. Moving load has constant direction and magnitude *F* = 100 *kN*. The load is moving along the beam with constant velocity *V*. In order to investigate the influence of the moving load velocity on the response of the curved beam, the displacement components at the free end were calculated. The calculations have been conducted using the isogeometric beam model with the 7th order B-Spline basis function (46 DOFs) obtained using the P-refinement procedure.

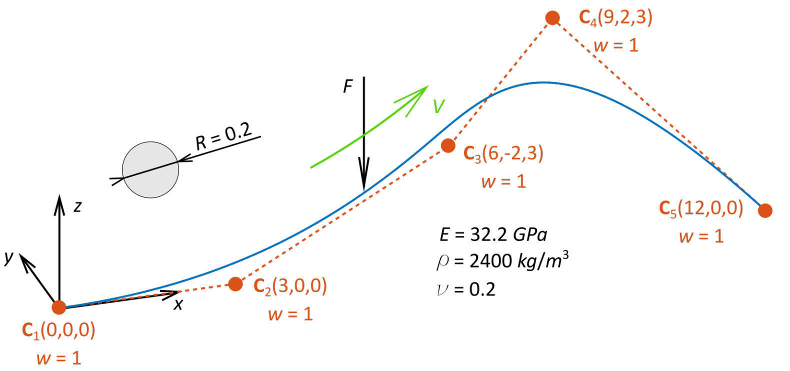

Figure 5. Cantilever spatial beam subjected to moving load

To compare the displacement components of the beam at the free end, the traveling time of the moving load has been divided by the total traveling time, forming a normalized dimensionless time coordinate, *t*[-]. Components of the beam displacements at the free end are presented in Figures 6 – 8. As the linear relation between the moving load magnitude and beam response holds for the linear dynamic formulation, the components of displacement are divided by the moving load magnitude, forming normalized displacements. Maximum values for *u* and *w* displacement components were detected for the moving load velocity of *V* = 9.25 m/s, while the maximum displacement component *v* occurred for the velocity of *V* = 22.5 m/s. In addition, the influence line has been calculated. The difference between the displacement components obtained using static analysis and dynamic analysis in case of the load velocity *V* = 1 m/s is not significant. However, the differences between displacement components increase as the load velocity increases, which can be observed in the case of the *w* displacement component. In addition, in the case of *u* and *v* displacement components, the moving load velocity can also affect the sign of the displacement components.

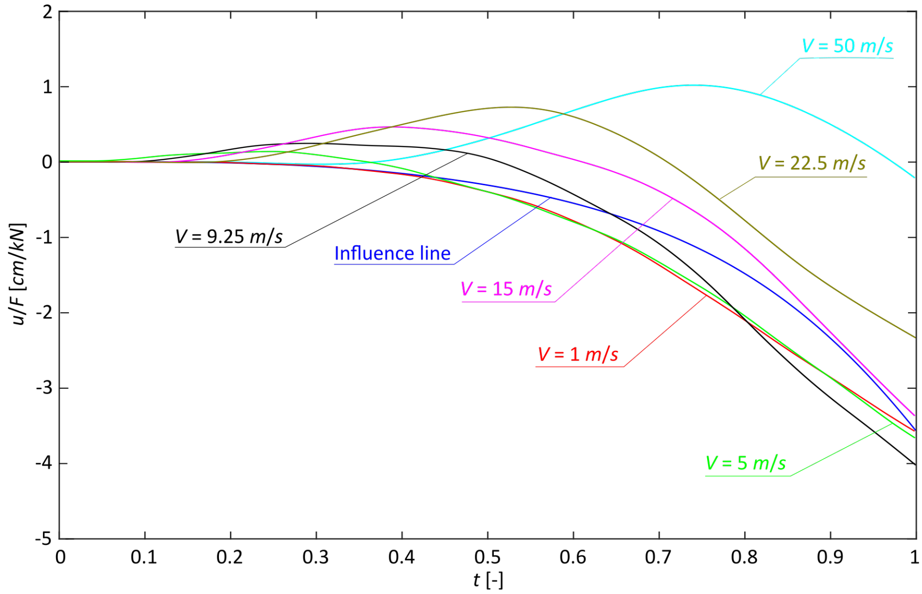

Figure 6. Normalized displacement component u with respect to the moving load velocity

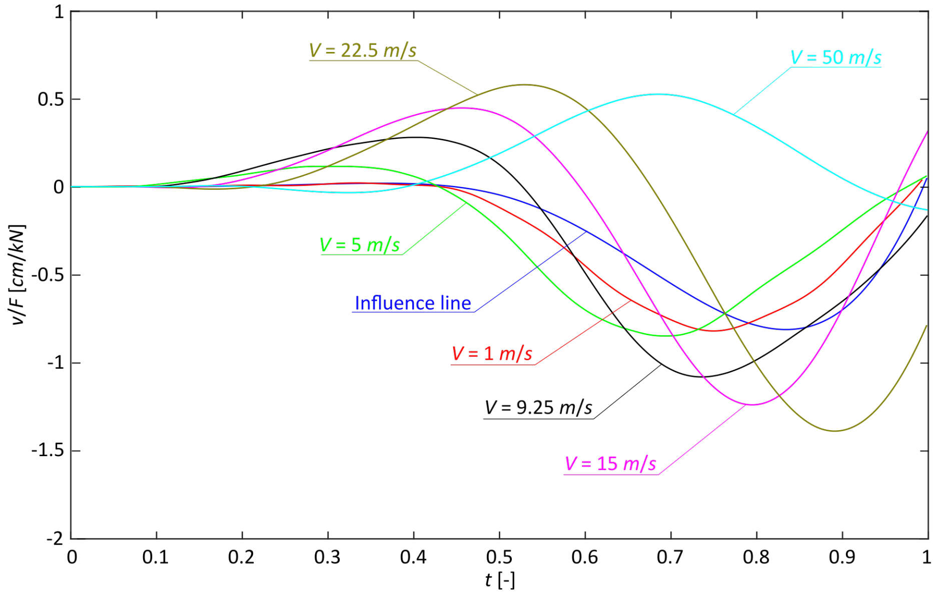

Figure 7. Normalized displacement component v with respect to the moving load velocity

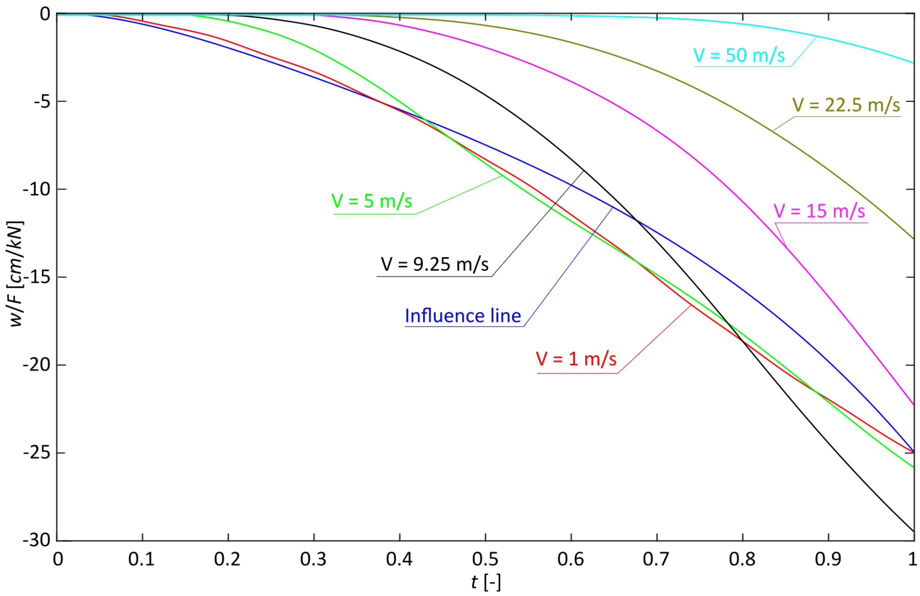

Figure 8. Normalized displacement component w with respect to the moving load velocity

7. CONCLUSIONS

The linear dynamic analysis of a spatially curved Bernoulli-Euler beam subjected to a moving load is presented. Spatial discretization is performed by the isogeometric approach, while the explicit procedure is used for the time integration. To validate the proposed method, the numerical study of the curved spatial beam subjected to the point load has been carried out. A satisfactory agreement has been observed between the results obtained using the proposed method and the results from the literature. In addition, the influence line for the displacement of the beam midpoint has been calculated, and the difference between the static and dynamic results is shown.

The influence of the moving load velocity on the arbitrary curved spatial beam has been investigated. It can be observed that the maximum displacement has occurred for the specific moving load velocity (critical velocity), and it is not the same for all beam displacement components. Also, the moving load velocity can affect the sign of the displacement. The accurate modeling of the moving load is crucial for the dynamic analysis of engineering structures such as bridges. In future work, more accurate models will be studied, taking into consideration the inertial part of the moving load. In addition, a nonlinear analysis, implicit procedures, and effects of the higher-order metric will be considered as well [14, 15].

ACKNOWLEDGEMENT

M. Jočković and M. Nefovska-Danilović are grateful to the Ministry of Science and Technology of the Republic of Serbia for the financial support of this research within the Projects TR-200092.

A. Borković acknowledges the support of the Austrian Science Fund (FWF): M 2806-N.

8. REFERENCES

[1] G. G. Stokes, "Discussion of a Differential Equation relating to the Breaking of Railway Bridges" in Mathematical and Physical Papers, vol. 2, Cambridge: Cambridge University Press, pp. 178–220, 2009.

[2] T. J. R. Hughes, J. A. Cottrell, Y. Bazilevs, „Isogeometric analysis: CAD, finite elements, NURBS, exact geometry and mesh refinement", Comput. Methods Appl. Mech. Eng. 194 (39–41), pp. 4135–4195, 2005.

[3] M. Jočković, G. Radenković, M. Nefovska – Danilović, M. Baitsch, "Free vibration analysis of spatial Bernoulli–Euler and Rayleigh curved beams using isogeometric approach", Appl. Math. Model. 71, pp. 152–172, 2019.

[4] L. Piegl, W. Tiller, *The Nurbs Book*, Springer, 1997.

[5] A. Borković, S. Kovačević, G. Radenković, S. Milovanović, M. Guzijan-Dilber, "Rotation-free isogeometric analysis of an arbitrarily curved plane Bernoulli–Euler beam", Computer Methods in Applied Mechanics and Engineering, 334, pp. 238-267, 2018.

[6] A. Borković, S. Kovačević, G. Radenković, S. Milovanović, D. Majstorović, "Rotation-free isogeometric dynamic analysis of an arbitrarily curved plane Bernoulli–Euler beam", Engineering Structures, 181, pp. 192-215, 2019.

[7] M. P. Do Carmo, *Differential Geometry of Curves and Surfaces*, Prentice-Hall, 1976.

[8] G. Radenković, *Finite rotation and finite deformation isogeometric structural analysis (in Serbian)*, University of Belgrade, Faculty of Architecture, 2017.

[9] G. Radenković, A. Borković, "Linear static isogeometric analysis of an arbitrarily curved spatial Bernoulli-Euler beam", Computer Methods in Applied Mechanics and Engineering, 341, pp. 360-396, 2018.

[10] M. Jočković, M. Nefovska – Danilović, "Isogeometric – based dynamic analysis of Bernoulli – Euler curved beam subjected to moving load", in *Proc. STEPGRAD XIV*, 2020, pp. 63-70.

[11] C. Adam, T. J. R. Hughes, S. Bouabdallah, M. Zarroug, H. Maitournam, "Selective and reduced numerical integrations for NURBS – based isogeometric analysis", Comput. Methods Appl. Mech. Eng. 284, pp. 732 – 761, 2015.

[12] MathWorks, MATLAB 2013, 2013. https://www.mathworks.com/products/matlab.html

[13] Y. B. Yang, C.-M. Wu,J.-D. Yau, "Dynamic Response of a Horizontally Curved Beam Subjected To Vertical and Horizontal Moving Loads", Journal of Sound and Vibration 242.3, pp. 519–537, 2001.

[14] A. Borković, B. Marussig, G. Radenković, "Geometrically exact static isogeometric analysis of an arbitrarily curved spatial Bernoulli–Euler beam", Computer Methods in Applied Mechanics and Engineering, 390, 114447, 2022.

[15] A. Borković, B. Marussig, G. Radenković, "Geometrically exact static isogeometric analysis of arbitrarily curved plane Bernoulli–Euler beam", Thin-Walled Structures, 170, 108539, 2022.

AUTHOR'S BIOGRAPHIES

Miloš Jočković

Miloš Jočković was born on April 6th 1988 in Sremska Mitrovica, Republic of Serbia. He defended his PhD thesis, "Isogeometric approach in dynamic analysis of spatial curved beam", in 2021 at the Faculty of Civil Engineering, University of Belgrade, where he is employed as an assistant professor.

Since 2013 he has been working as a researcher in the Project "Towards Development of Sustainable Cities: Influence of traffic-induced vibrations on buildings and humans". During 2013 he participated in the Project "Implementing MATLAB tool for analysing flutter instability". Since 2014 he has been working at the Faculty of Civil Engineering, University of Belgrade, as a Teaching Assistant - PhD Student in the scientific field of Engineering Mechanics and Theory of Structures.

His main scientific interests focus on structural dynamics, nonlinear analysis, isogeometric analysis and finite element method.

Marija Nefovska-Danilović

Marija Nefovska-Danilović was born on October 9th 1972, in Skopje, Republic of North Macedonia. She obtained her PhD in the field of Engineering Mechanics and Theory of Structures in 2013. Since 1997 she has been employed at the Faculty of Civil Engineering, University of Belgrade. Since 2018 she has been appointed as Associate Professor.

Her main research expertise includes vibration of plates and shells, dynamic stiffness method, dynamic soil-structure interaction, isogeometric analysis, vibration of multilayer composites and vibration serviceability of civil engineering structures.

Aleksandar Borković

Aleksandar Borković was born on January 12th, 1982 in Gradiška. He obtained his PhD in the field of Civil Engineering in 2014, and has been appointed as Associate Professor since 2019 at the Faculty of Architecture, Civil Engineering and Geodesy, University of Banja Luka. Since 2020, he has been a Researcher at the Institute for Applied Mechanics, Graz University of Technology.

The focus of his research are numerical, analytical and experimental methods for the analysis of engineering structures.

ЛИНЕАРНА ДИНАМИЧКА АНАЛИЗА УТИЦАЈА ПОКРЕТНОГ ОПТЕРЕЋЕЊА НА ПРОСТОРНОЈ КРИВОЛИНИЈСКОЈ БЕРНУЛИ-ОЈЛЕРОВОЈ ГРЕДИ

Сажетак: У овом раду је приказана динамичка анализа просторне криволинијске Бернули-Ојлерове греде под утицајем покретног оптерећења. Изогеометријски приступ је примењен у циљу просторне дискретизације слабе форме једначина кретања греде. Овај приступ се базира на примени истих базних NURBS функција за описивање геометрије и кинематике криволинијске греде, чиме је омогућен тачан приказ системне линије греде. Временска интеграција једначина је извршена применом експлицитне методе. Приказана формулација је валидирана поређењем са резултатима из литературе за случај криволинијске греде оптерећене покретном силом константног интензитета и брзине. Такође је извршена и анализа утицаја брзине кретања покрене силе на динамички одговор просторне криволинијске греде.

Кључне ријечи: изогеометријски приступ, Бернули-Ојлерова крива греда, покретна сила