Aerial view of tunnel with trains - Erie Railway, Otisville Tunnel, Sanitarium Road to Otisville Road, Otisville, Orange County, NY. Photographer: Boucher, Jack, 1968. Source: https://www.loc.gov/pictures/item/ny1220.photos.121395p/ (Wikimedia Commons)

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ON SIMPLIFIED APPROACHES OF SEISMIC ANALYSIS OF TUNNELS

ABSTRACT

Overview of current progress in the field of seismic regulations for the design of tunnel structures revealed that, despite significant progress in research work on seismic analysis of tunnels over the past few decades, however, a deficiency of systematic and precisely defined rules for the seismic design of tunnels still exists even in the most developed societies. Precisely for this reason, a great effort has recently been made in this research field in terms of finding simple approaches of the seismic analysis of tunnels that could be implemented in design codes and thus serve designers in everyday engineering practice. The response of tunnel structures to earthquake excitation is primarily conditioned by the strain field in the surrounding ground. The simplest approach in seismic analysis of tunnels is based on the assumption that deformations in the circular tunnel are identical to the deformations of the ground induced by seismic waves in its natural state, without tunnel excavation (the so-called "free-field deformation approach"). In addition, seismic design of tunnel structures taking into account the effects of soil–structure interaction is becoming increasingly important nowadays, because the effects of the interaction between the structure and the surrounding gorund can cause greater external forces on the tunnel structure (the so-called "soil–structure interaction approach"). The present study considers the most frequently used simple analytical expressions, regarding the idealised tunnel geometry and ground properties, for calculating the relevant design soil shear strain that occurs between the depths that correspond to the tunnel crown and the invert, on the one hand, and for determining the seismically induced forces in the tunnel lining taking into account the soil–structure interaction effects, on the other hand. Furthermore, in order to evaluate the ability of the analytical expressions to simulate the most important aspects of the seismic behaviour of tunnels, numerical analyses were also carried out by one-dimensional free-field ground response analysis in the code EERA and by the simplified dynamic soil–structure interaction analysis in the software ANSYS, respectively. Lastly, the results obtained by the simple analytical and numerical approaches were evaluated, considering the main soil types – stiff soil with good properties and soft saturated soil with poor properties.

Keywords: *circular tunnel, earthquake, design codes, seismic analysis, simplified approaches*

1. INTRODUCTION

Traffic infrastructure, of which tunnels are integral parts, is considered of great significance when considering the risk of strong earthquakes. The availability of roads affects the speed and extent of emergency measures to be taken in emergency and relief operations immediately after an earthquake. Furthermore, earthquake-induced damage to infrastructure may seriously affect the earthquake-affected region's economy due to the time required to restore network functionality. In addition, underground structures are often located beneath densely populated urban areas. Considering all abovementioned facts, tunnel structures require very high standards regarding their stability, safety, and reliability [1]. In this regard, in the following part, a brief overview of the current progress in the field of seismic regulations for the design of tunnel structures is presented.

After the Hyogoken Nanbu (Kobe) Earthquake in 1995, which was the first case of serious damage to modern underground structures caused by an earthquake**, earthquake-resistant design regulations in Japan** were revised by defining two levels of design: low-to-moderate earthquakes and strong earthquakes. In the "Standard Specifications for Tunneling" [2], published by the Japan Society of Civil Engineers, mountain tunnels, shield tunnels, and cut-and-cover tunnels are discussed. According to "Standard Specifications for Concrete Structures - Design" [3], it is recommended to consider the usage of structures and materials designed to increase flexibility, with an aim to maintain the required seismic behaviour of underground structures. Therefore, especially for the seismic analysis of shield tunnels, based on the seismic deformation method, calculation approaches using the bedded-beam model [4] with appropriate ground springs and structural joint- springs are proposed with the use of elastic analysis.

Despite the fact that **seismic design regulations in the United States of America** are highly developed, there is still a lack of adequate codes in the field of seismic design of tunnels. Recommendations of the American Society of Civil Engineers provided within the code ASCE/SEI 7-10 “Minimum Design Loads for Buildings and Other Structures” [5], are not dealing with tunnel structures (Chapter 15 "Seismic design requirements for non-building structures" states that underground lines and their appurtenances are not included in the scope of requirements for non-building structures). For tunnel structures, Chapter 13 of the "Technical Manual for the Design and Construction of Road Tunnels" [6], proposed by the Federal Highway Administration, provides good practice. It provides a general procedure for the seismic design and analysis of underground structures based primarily on the ground deformation method, which is the opposite of the inertial force approach typical for aboveground structures. Consequently, tunnel structures should be designed to conform the surrounding ground deformations. Yet, this procedure is only a recommendation, it is not a standard or regulation.

Standards for the seismic design of structures in the countries of the European Union are presented within Eurocode 8. The European Standard EN 1998-4 “Eurocode 8: Design of structures for earthquake resistance – Part 4: Silos, tanks, and pipelines” specifies the principles and rules for the seismic design of aboveground and underground pipeline systems, storage tanks, and silos of different types and uses [7]. Moreover, in the European Standard EN 1998-5 “Eurocode 8: Design of structures for earthquake resistance: Foundations, retaining structures, and geotechnical aspects” [8], as part 5 of the European Seismic Regulation, requirements, criteria, and rules are defined for the design of various earthquake-resistant foundation systems and retaining structures, as well as for the seismically induced soil–structure interaction. However, provisions related to the seismic design of tunnels are not provided for in these standards.

The **seismic design code in the Russian Federation** SP14.13330.2014 [9] are the newest version of the previous seismic design code SniP II-7-81. Unlike European standards, it represent a single document embracing everything from foundation structures to fire protection. Section 7.9 of the code is dedicated to tunnel structures, recommending the application of the corresponding type of tunnel lining depending on the level of seismicity, as well as the use of anti-seismic expansion joints. Given the calculation procedure, in Section 8.4 the effect of earthquakes is defined to some extent through the appropriate dynamic coefficients.

Design standards in the Republic of Serbia are prepared in accordance with the aforementioned European norms and accompanying documents. In the field of seismic design, there are SRPS EN 1998-4 [10] and SRPS EN 1998-5 [11], and they are related to the corresponding European norms. Consequently, as in the case of Eurocode 8, the SRPS standards and guidelines do not specifically deal with the issue of seismic design of underground structures. The "Collection of Yugoslav regulations and standards for engineering constructions" [12] was previously published, in which a draft version of the "Regulations on technical rules for the design and calculation of engineering structures in seismic areas" was created as part of the "Actions on structures" section. This version of the regulations provided a methodology for determining the seismic pressure of the ground on underground and buried structures. It was the beginning of raising awareness about the importance of aseismic design of tunnel structures, as well as the beginning of placing this issue within the framework of standards. Despite this concept, which at that time represented a great advance in the standardisation practice, unfortunately, this draft version remained at the level of ideas and proposals and never entered into force.

On the basis of the presented short review on the current standards and codes for aseismic design of structures, it can be concluded that there is a deficiency of systematic and precisely defined rules for the seismic design of tunnels. It is obvious that even in the most developed societies there is a noticeable discrepancy between the currently valid regulations for tunnel structures, especially with regard to earthquake activity, and the requirements for the design and construction of safe and cost-effective underground structures. Moreover, considering twin-tunnels, it should be noted that research on the mutual influence of closely located tunnels, where the aspect of their minimum seismically safe distance should be of utmost importance, is still at an initial level [13]–[15]. Accordingly, it should be said that we have a serious task ahead of us. This study attempted to improve this situation, as it deals with the review and evaluation of simple approaches to the seismic analysis of single tunnels that could be considered in seismic design codes for tunnels and thus serve in daily engineering practice.

2. METHODS OF SEISMIC RESPONSE ANALYSIS OF TUNNELS

Tunnel structures have characteristics by which their seismic behaviour differs from most aboveground structures, such as their complete constraint by the surrounding medium (soil or rock) and their considerable length. Aboveground structures are designed to accommodate the inertial forces induced by ground accelerations with focus on inertial effects of the structure itself (Seismic Force Method), which is completely opposite to the design of tunnels, in which case seismic design loads are defined by stresses and strains imposed on the tunnel structure by the surrounding ground (Seismic Deformation Method).

The seismic response of underground structures may be assessed using two approaches: the "*free-field deformation approach*" and the "*soil–structure interaction approach*" [16]. These two approaches include different sub-methods with different levels of approximation, which depends on the design stage, as well as knowledge of geological conditions and geotechnical parameters. Regarding the types of analyses, they may be grouped into three categories: "*pseudo-static*", "*simplified dynamic*", and "*full (detailed) dynamic analysis*", depending on the desired level of complexity related to the selected model, soil characterisation, and seismic input description.

2.1. FREE-FIELD DEFORMATION APPROACH

The simplest approach is the so-called the "free-field ground deformation" approach. The term "free-field deformation" refers to ground deformations caused by seismic waves in the absence of tunnel excavations or structures, meaning that this approach does not take into consideration the interaction between the underground structure and the surrounding ground. However, it can provide a simple and fast first-order estimate of the predicted structure deformation. So the essence of the procedure is that free-field ground deformations due to the seismic event are evaluated and the underground structure is designed to accommodate these deformations.

Given the level of approximation, deformations of the structure using this approach may be overestimated or underestimated, which primarily depends on the stiffness of the structure relative to the ground stiffness. The results are satisfactory for the cases of low levels of shaking, the tunnel structure in a rock medium, or when the tunnel structure is flexible in comparison with the surrounding ground (such as the case of a tunnel in a rock medium, in which case the stiff surrounding ground deformations cannot be affected considerably by the stiffness of the structure). Yet, in many other cases, particularly in the case of soft soils, this method yields conservative design, since free-field deformations in soft soils are in general quite large.

2.1.1. Closed-form elastic solutions for circular tunnels (pseudo-static analysis)

Simplified closed-form elastic solutions are fruitful for obtaining an initial assessment of tunnel deformations. These simplified methods are based on the assumption that the seismic wave field is a field of plane waves, which have the same amplitudes at all locations along the tunnel's length and differ only in their arrival time. Thus, the complex three-dimensional wave propagation and scattering that lead to differences in wave amplitudes along the tunnel's length are not taken into consideration, although this ground motion incoherence may enlarge stresses and deformations in the tunnel's longitudinal direction. Therefore, the results of analyses based on the plane wave propagation assumptions have to be interpreted with a great caution.

The component that has the most significant effect upon the tunnel lining under the action of earthquakes is the ovaling deformation (ovalisation), with vertically propagating shear S-waves being predominant form of seismic loading that causes these types of deformations. The results of the ovaling deformation are cycles of additional stress concentrations with alternating compressive and tensile stresses in the tunnel lining, whereby the following critical modes are possible:

- compressive dynamic stresses added to the compressive static stresses can locally exceed the compressive strength of the tunnel lining;
- tensile dynamic stresses subtracted from compressive static stresses can locally reduce the bending strength of the tunnel lining, with a tendency for the resulting stresses to be tensile.

The ovalisation is usually simulated under the two-dimensional plane strain condition. The resulting free-field ground shear distortion can be expressed as a shear distribution, i.e. a shear strain profile as a function of depth. The simplest way of ovaling deformation estimation is based on the assumption that the deformations in the circular tunnel are identical to the free-field ground deformations, thus neglecting the soil–tunnel structure interaction. The circular tunnel–ground shearing can be modelled in two ways [16]:

1) As a continuous medium without the presence of a tunnel (i.e. non-perforated ground presented in Fig. 1(a)), whereby the circular tunnel distortion or diametric strain can be calculated as:

$$\frac{\Delta d_{free-field}}{d} = \pm\frac{\gamma_{max}}{2} \tag{1}$$

where γ_{max} is the maximum free-field ground shear strain and d is a diameter of the tunnel. It is obvious that, in this case, the maximum diametric strain of the circular section is solely a function of the maximum free-field shear strain. This assumption is reasonable in the case when the ovaling stiffness (i.e. stiffness against distortion) of the lined tunnel is identical to the stiffness of the surrounding ground.

2) The circular tunnel distortion or diametric strain is calculated under the assumption of an unlined tunnel (i.e. perforated ground in Fig. 1 (b)):

$$\frac{\Delta d_{free-field}}{d} = 2\gamma_{max}(1-\nu_{gr}) \tag{2}$$

where ν_{gr} is Poisson ratio of the ground. In this case, apart from the maximum free-field shear strain, the maximum diametric strain is related to the Poisson ratio of the ground as well. This assumption is convenient in the case when the ovaling stiffness of the lined tunnel is very small compared to the surrounding ground, i.e. for circular tunnels in rock media or stiff soils.

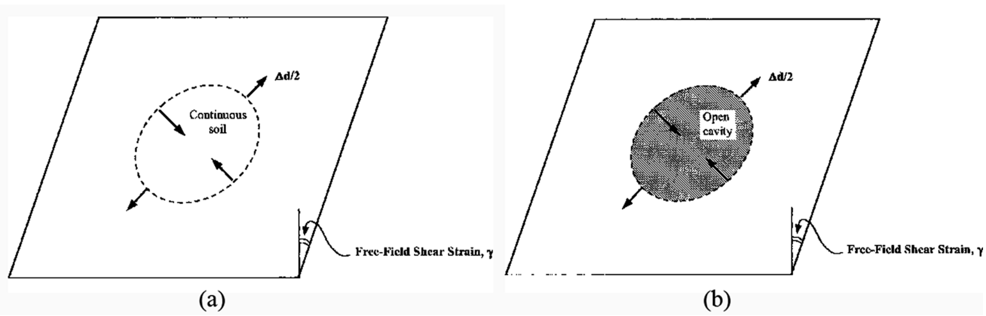

Figure 1. *Free-field shear distortion (circular shape): (a) non-perforated ground; (b) perforated ground [16]*

Both Eq. (1) and Eq. (2) assume the non-existence of the tunnel lining, thus neglecting the tunnel–ground interaction. In a free field, the perforated ground will reach a much higher distortion than the non-perforated ground, with distortion sometimes two or three times greater. This is an appropriate distortion criterion for a tunnel lining of a lower stiffness in comparison with that of the surrounding ground. The deformation equation for the non-perforated ground, on the other hand, will be reasonable when the stiffness of the tunnel lining is identical to that of the surrounding ground. A tunnel lining with stiffness higher than that of the surrounding ground (i.e., when a tunnel is built in a soft or very soft soil) will experience distortions less than those given by Eq. (1).

2.1.2. Earthquake-induced maximum soil shear strain γ_{max}

Considering the fact that the transient deformation of the soil during the action of an earthquake cannot be measured directly, it is common practice to indirectly calculate the peak deformations of the soil using simplified expressions derived under the assumption of propagation of plane waves in a homogeneous medium. The maximum shear strain in the free field is expressed as [16]:

$$\gamma_{max} = \frac{V_{S,max}}{C_S} \tag{3}$$

where $V_{S,max}$ is the peak particle (peak ground) velocity associated with shear S-wave and C_S is the apparent (effective) shear wave propagation velocity (i.e., maximum mass velocity in the ground). The values of C_S can be obtained on the basis of *in situ* and laboratory tests. The effective shear wave propagation velocity is related to the effective shear modulus of the ground G_{gr} according to the following expression:

$$C_S = \sqrt{\frac{G_{gr}}{\rho_{gr}}} \tag{4}$$

where ρ_{gr} is the mass density of the ground.

The expression in Eq. (3) has an extensive application in engineering practice, since it enables a simple estimation of design stresses. Despite its simplicity, a number of input quantities that are not easy to determine are however required, such as incidence angle, apparent wave propagation velocity, predominant wave type, etc. Accordingly, it can be used only in situations where the assumptions of its derivation are met (e.g., one-dimensional plane harmonic propagation of waves in a homogeneous medium). In addition to wave propagation characteristics, there are also effects that are not taken into account in this expression, such as spatial incoherence, site effects, as well as near-fault effects.

2.1.3. Relevant earthquake-induced soil shear strain at the tunnel depth γ_{rel}

In the seismic analysis of tunnel structures, the peak ground strain during an earthquake is not relevant for the appearance of pressures on the tunnel structure, but the soil shear strain occurring between the depths associated with the tunnel crown and the invert. Data on strong ground motion at the depths of concern for tunnel structures are usually not available. Accordingly, the design ground motions include depth-dependent attenuation effects (i.e., ground motion generally decreases with depth). Table 1 and the expression in Eq. (5) can be used to find a relationship between the ground motion at ground surface and the ground motion at the corresponding depth:

Table 1. *Ratios of ground motion at depth to motion at the ground surface [17]*

Tunnel depth [m]	**Ratio of ground motion at a tunnel depth to motion at the ground surface**
≤ 6	1.0
6–15	0.9
15–30	0.8
> 30	0.7

$$a_{S,depth} = (\text{coefficient from Table 1}) \cdot \ a_{S,max} \tag{5}$$

where $a_{S,max}$ is the peak particle (peak ground) acceleration.

Earthquake-induced damage to tunnel structures is primarily correlated with particle velocity and displacement, not acceleration. Existing attenuation relationships are usually applicable to estimate peak ground surface acceleration; however, they can also be used to estimate peak ground velocity and displacement. In the case where site-specific data are not available, based on Tables 2 and 3 and the known peak ground acceleration, the peak velocity and displacement can be obtained, respectively. In all the presented tables, the types of sediments represent the following shear wave velocity ranges: in rock medium $C_S \geq 750$ m/s, in stiff soil C_S = 200–750 m/s, and in soft soil $C_S < 200$ m/s. It should be noted that the given ratios of peak ground velocity to peak ground acceleration are less certain with regard to soft soils [17].

Table 2. *Ratios of peak ground velocity to peak ground acceleration at surface in rock and soil [17]*

Moment magnitude Mw	***Ratio of peak ground velocity [cm/s] to peak ground acceleration [g]***		
	Distance from source to site[km]		
	0–20	**20–50**	**50–100**
Rock			
6.5	66	76	86
7.5	97	109	97
8.5	127	140	152
Stiff soil			
6.5	94	102	109
7.5	140	127	155
8.5	180	188	193
Soft soil			
6.5	140	132	142
7.5	208	165	201
8.5	269	244	251

Accordingly, the particle (ground) velocity at the tunnel depth would be:

$$V_{S,depth} = (\text{value from Table 2 [cm/s/g]}) \cdot \ a_{S,depth} \ [\text{g}] \tag{6}$$

Table 3. *Ratios of peak ground displacement to peak ground acceleration at surface in rock and soil [17]*

Moment magnitude Mw	***Ratio of peak ground displacement [cm] to peak ground acceleration [g]***		
	Distance from source to site [km]		
	0–20	**20–50**	**50–100**
Rock			
6.5	18	23	30
7.5	43	56	69
8.5	81	99	119
Stiff soil			
6.5	35	41	48
7.5	89	99	112
8.5	165	178	191
Soft soil			
6.5	71	74	76
7.5	178	178	178
8.5	330	320	305

Analogously, the particle (ground) displacement at the tunnel depth is calculated by the following formula:

$$D_{S,depth} = (\text{value from Table 3 [cm/g]}) \cdot \ a_{S,depth} \text{ [g]} \tag{7}$$

Finally, the relevant soil shear strain γ_{rel} at the depth of the longitudinal central axis of the tunnel, induced by the propagation of seismic shear S-waves, is given by the following expression:

$$\gamma_{rel} = \frac{V_{S,depth}}{C_S} \tag{8}$$

2.1.4. One-dimensional seismic site response analysis (simplified dynamic analysis)

This method aims to calculate the earthquake-induced acceleration, shear stress, strain, and maximum ground displacements in a range of depths related to the tunnel section, between the tunnel crown and the invert, using a one-dimensional (1D) free-field seismic site response (SSR) analysis. In doing so, both the time history of the acceleration and the site characteristics are taken into consideration, whereas the effects of the tunnel–ground interaction are still not taken into account.

Seismic waves, emanating from the source, may travel even tens of kilometers through the rock medium and usually less than 100 m through the overlying soil. Nevertheless, soil can contribute significantly to the ground surface motion characteristics. A major issue in the ground response analysis is determination of the response of the soil deposit to the motion of the underlying bedrock. As presented in Fig. 2(a), the motion of the surface of the soil deposit is called the *free surface motion*, the motion of the base of the soil deposit (which at the same time represents the motion of the top of the bedrock) is called the *bedrock motion*, whereas the motion of the bedrock exposed at the ground surface is called the *rock outcropping motion*. In the case when the soil deposit does not exist (as presented in Fig. 2(b)), the motion of the top of the bedrock is called the *bedrock outcropping motion* [18].

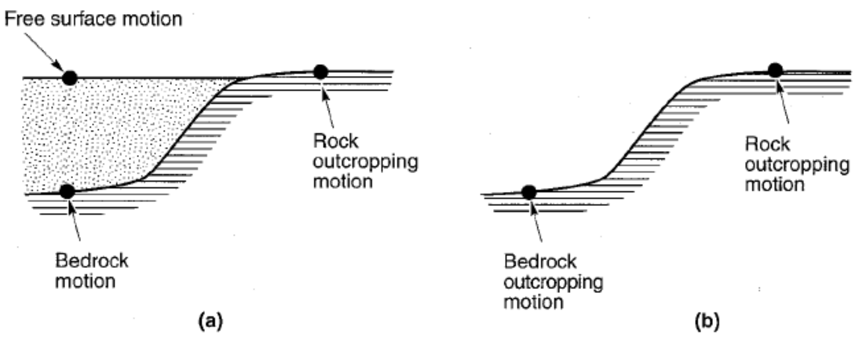

Figure 2. *Nomenclature in the ground response analysis: (a) soil deposit overlying bedrock; (b) no soil deposit overlying bedrock [18]*

After a fault occurs below the ground surface, seismic body waves travel away from the source in all directions. On their path to the ground surface, the waves reach boundaries between two geological materials of different characteristics and are being reflected and refracted. Considering that the propagation velocities of seismic waves in shallow soil media are mostly lower than the velocities in the underlying media of greater depths, inclined seismic waves that strike horizontal layer boundaries tend to be reflected in a more vertical direction. During the travel of seismic waves towards the ground surface, they are bent in an almost vertical direction (Fig. 3) due to manifold refractions. In one-dimensional ground response analyses, all boundaries are assumed to extend infinitely in the horizontal direction (typically a distance several times the total depth to the bedrock) and the response of soil deposits is assumed to be primarily induced by vertically propagating S-waves (SH- or SV-waves) travelling from the underlying bedrock.

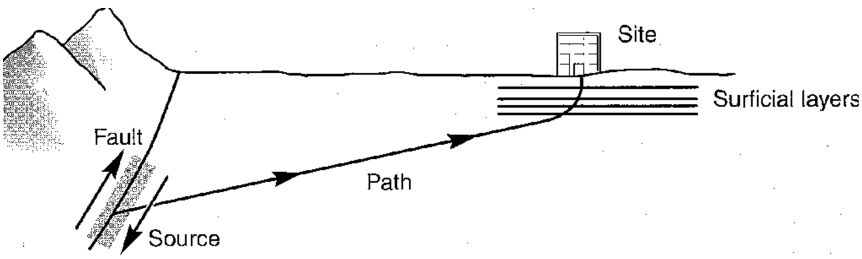

Figure 3. *Multiple refraction of seismic waves that produces near-vertical wave propagation near the ground surface [18]*

Many computer programs are available for 1D wave propagation analysis: SHAKE [19], EERA - Equivalent-linear Earthquake site Response Analysis [20], NERA - Nonlinear Earthquake site Response Analysis [21], DEEPSOIL [22], SPECFEM 1D [23] based on the Spectral Element method (SEM).

Ground response analysis is based on the application of the so-called transfer functions, by the virtue of which a variety of output quantities (i.e., response parameters, such as displacement, velocity, acceleration, shear stress, and shear strain) can be related to an input motion parameter (i.e., bedrock acceleration). This approach relies on the principle of superposition, and therefore it is reasonable to be used exclusively in linear analysis. The principle is as follows:

- The known input quantity (time history of bedrock motion) is represented by a Fourier series, using the Fast Fourier Transform (FFT).
- Each term in the Fourier series of the bedrock motion is being multiplied by the transfer function in order to obtain the output quantity (Fourier series of the ground surface motion).
- The ground surface motion is being expressed in the time domain based on the inverse Fast Fourier Transform.
- Based on this, the transfer function defines for each frequency in the bedrock input motion whether it is amplified or deamplified owing to the presence of the soil deposit.

2.2. SOIL–STRUCTURE INTERACTION APPROACH

The soil–structure interaction (SSI) effects have recently become an indispensable part of the analysis and design of tunnels under earthquake conditions, because it has been proven that the effects of the interaction between the tunnel structure and the surrounding ground may result in greater external forces acting on the structure. The presence of the structure considerably modifies the free-field ground motion, leading to a different seismic response of the tunnel lining. The interaction effects are manifested in the form of kinematic interaction and inertial interaction, which most often act in combination. The kinematic interaction occurs due to the inability of the tunnel to follow ground motion due to its higher stiffness compared to ground stiffness and has been proven to be of primary importance. The inertial interaction is often considered less important and could be ignored as the tunnel structure inertia is negligible compared to the surrounding ground inertia [25].

Tunnel–ground interaction under seismic action is considerably more complex compared to that of aboveground structures. In the case of aboveground structures only foundations are exposed to the soil–structure interaction effects, whereby the vibrations of the ground particles imposed to the foundations are being transmitted to the structure above the ground surface. When it comes to tunnel structures, on the other hand, the soil–structure interaction is induced along the entire structural contour, whereby the form of interaction depends primarily on the type of construction procedure, that is, on the excavation methodology and installation technology of the tunnel support system. The effect of an earthquake on the tunnel–ground interaction depends on a number of parameters, such as the peak ground acceleration, the intensity and duration of the earthquake, and the relative stiffness between the tunnel and the surrounding ground. Thus, in the case of a rigid liner in a soft soil, the soil cannot produce tunnel deformation; however, in the case of a flexible liner, there is an interaction between the liner and the surrounding soil.

There are a number of approaches that allow dynamic soil–structure interaction to be taken into account when designing a tunnel structure. In these approaches, for simplicity, it is assumed that the soil behaves as a linear elastic or viscoelastic material and is perfectly connected to the tunnel structure (the so-called no-slip condition). However, in reality, the bond between the soil and the structure is rarely of a perfect nature, as slippage or even separation in the contact surface may occur (the so-called full slip condition), which is especially typical in the case of tunnels in very soft soils or under strong earthquakes. In most cases, there is a partial slip condition (owing to large deformations of the soil, the soil–structure interaction decreases with increasing relative displacements between the soil and the structure). Therefore, it is always recommended to consider both extreme cases (no-slip condition and full slip condition) and apply the more critical one.

2.2.1. Simplified analytical SSI approach for circular tunnels (pseudo-static analysis)

In pseudo-static approach, tunnel and soil analysis are separated. The seismic input is represented by the peak strain amplitude. This quantity is calculated according to simplified formulae based on the simple assumption of the propagation of plane harmonic S-waves in a homogeneous, isotropic, elastic medium. After that, its action on the tunnel lining under static conditions is considered. In doing so, the influence of the shape and stiffness of the tunnel on the seismic behaviour of the ground is not taken into account.

The simplified analytical approach, proposed by Wang [25], is based on the theory of an elastic beam on an elastic foundation, by which the effects of the tunnel–ground interaction are considered under quasi-static conditions. The solution refers to circular tunnels, the most critical deformation pattern of which is ovalisation (distortion, shearing) of the circular cross-section of the tunnel, caused by shear S-waves that propagate in planes perpendicular to the longitudinal axis of the tunnel. Ovalisation is usually simulated under the two-dimensional plane-strain condition. Such an approach is justified for the following reasons: (1) the typical cross-sectional dimensions of the tunnel liner are small compared to the wavelengths of the predominant ground motion that induces the ovaling deformation; (2) the effects of inertia in the tunnel lining and the surrounding soil as a result of the dynamic effects of the interaction between the soil and the structure are relatively small. Furthermore, the solution is based on the assumption that the soil behaves in a linear elastic manner. For the case of *no-slip* tunnel–ground interface condition (perfect bond, rigid contact, or rough interface) that considers the continuity of stresses and displacements and no relative shear displacements of the ground and tunnel liner at the common interface, the expressions for the bending moment *M* and thrust (axial force) *T* (Fig. 4) according to the Wang's solution, in terms of an angle θ measured counterclockwise with respect to the axis of the tunnel spring line, are:

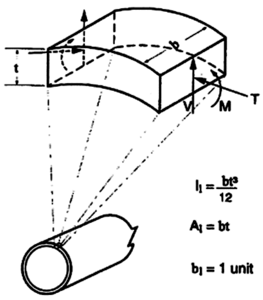

Figure 4. *Circumferential tunnel lining forces and moments induced by seismic waves propagating perpendicular to tunnel longitudinal axis [16]*

$$M(\theta) = \pm\frac{1}{12}K_1 G_{gr} d^2 \cdot \gamma_{rel} \cos\left[2\left(\theta + \frac{\pi}{4}\right)\right] \Rightarrow M_{max} = \pm\frac{1}{6}K_1 \frac{E_{gr}}{1+\nu_{gr}} r^2 \cdot \gamma_{rel} \quad (9)$$

$$T(\theta) = \pm\frac{1}{2}K_2 G_{gr} d \cdot \gamma_{rel} \cos\left[2\left(\theta + \frac{\pi}{4}\right)\right] \Rightarrow T_{max} = \pm K_2 \frac{E_{gr}}{2(1+\nu_{gr})} r \cdot \gamma_{rel} \quad (10)$$

where:

$$K_1 = \frac{12(1-\nu_{gr})}{2F+5-6\nu_{gr}} \tag{11}$$

$$K_2 = 1 + \frac{F\left[(1-2\nu_{gr})-(1-2\nu_{gr})C\right]-\frac{1}{2}(1-2\nu_{gr})^2+2}{F\left[(3-2\nu_{gr})+(1-2\nu_{gr})C\right]+C\left(\frac{5}{2}-8\nu_{gr}+6{\nu_{gr}}^2\right)^2+6-8\nu_{gr}} \tag{12}$$

$$C = \frac{E_{gr}(1-{\nu_{lin}}^2)r}{E_{lin}t_{lin}(1+\nu_{gr})(1-2\nu_{gr})} \tag{13}$$

$$F = \frac{E_{gr}(1-{\nu_{lin}}^2)r^3}{6E_{lin}I_{lin}(1+\nu_{gr})} \tag{14}$$

$$\frac{\Delta d_{lin}}{d} = \pm\frac{1}{3}K_1F\cdot\gamma_{rel} \tag{15}$$

In the above given equations, r is the tunnel radius, d is the tunnel diameter, t_{lin} denotes the thickness of the tunnel lining, I_{lin} is moment of inertia of the tunnel lining (per unit width) for circular tunnel, ν_{lin} is the Poisson ratio of the lining, E_{lin} is the elasticity modulus of the lining, , $\Delta d_{lin}/d$ is the diametric strain of the lining, ν_{gr} is the Poisson ratio of the soil, E_{gr} is the elasticity modulus of the soil, $G_{gr} = \rho_{gr} \cdot C_S{}^2$ is the soil shear modulus (it relates the velocity of propagation of shear waves in the ground C_S to the mass density of the ground ρ_{gr}), γ_{rel} is the relevant free-field shear strain (i.e., the mean value of the free-field shear strain in the depth range corresponding to the tunnel crown and the invert), K_1 stands for the moment response coefficient, K_2 represents the thrust response coefficient.

In order to understand the significance of tunnel lining stiffness, there are two dimensionless parameters that relate the stiffness of the tunnel and the surrounding ground. The first is the compressibility ratio C, as a measure of the compressive stiffness of the ground in relation to the tunnel lining under the free-field uniform or symmetrical loading conditions (vertical soil stress = horizontal soil stress), and it reflects the circular stiffness of the tunnel–ground system (i.e., resistance to compression). The second is the flexibility ratio F, which is a measure of the shear stiffness of the ground with respect to the tunnel lining under the free-field antisymmetric loading condition (horizontal ground stress equal, but opposite in sign, to the vertical ground stress in the free-field), and it reflects the radial stiffness of the tunnel–ground system (i.e., resistance to ovalisation). Of these two ratios, the flexibility ratio is suggested to be more important, as it reflects the ability of the tunnel lining to resist shearing deformations imposed by the surrounding ground (for more details, see [26]).

The above presented analysis procedures can be reasonably applied to tunnel linings with sufficiently large burial depths (the so-called deeply embedded tunnels), so that the boundary conditions of free surface at the top of the soil and bedrock at the bottom of the soil, have a negligibly small effect on soil–structure interaction. As shown by Wang [25], these boundary effects could be considered negligible in the case of circular tunnel linings with a ratio $h/d > 1.5$ (where h is the distance from the free surface, as well as the bedrock, to the mid-height of the lining, and d is the outer diameter of the lining). Furthermore, these solutions are adequate for cast-in-place concrete tunnel linings or shield tunnel linings composed of prefabricated-concrete segments, i.e. for cases when the linings are placed in soils with lower values of the modulus of elasticity compared to the modulus of the tunnel lining, that is, for the case of linings in soft soils when the effects of soil–structure interaction are particularly pronounced.

Although the given solutions were proposed several decades ago, they are still the most commonly used analytical solutions today. The main reason lies in the fact that the problem of soil–tunnel structure interaction under the influence of earthquakes has not been fully investigated and known until now, and according to the author's knowledge, there has been no evident progress in this regard during the past decade or two. In addition, the literature dealing with the effect of earthquakes on tunnel structures is quite rare, and therefore presenting analytical solutions based on the theory of elasticity is fruitful, in order to see the basic assumptions and limitations that are essential in these solutions. Finally, advanced numerical analyses using contemporary software are rather complex and time-consuming, and are therefore focused on case-specific studies. The simplified approaches mentioned above, on the other hand, allow relatively quick and simple analysis and provide reasonable results for the needs of engineering practice.

2.2.2. Simplified dynamic SSI analysis

In a simplified dynamic SSI analysis, soil strains over a range of depths corresponding to the tunnel section (i.e., at depths between the tunnel crown and the invert) are calculated by one-dimensional (1D) free-field seismic site response (SSR) analysis and after that applied to the tunnel lining, under pseudo-static conditions as was the case with the previously explained simplified solution. By that, both the time history acceleration and the characteristics of the site are taken into account, but the kinematic soil–structure interaction is still not taken into account. Furthermore, the effects of compressional waves are also ignored, given that solely shear waves are considered, with propagation in vertical planes causing shear deformations.

Contemporary technological development has contributed to the development of a large number of software, which are based on the principles of the finite element method (FEM) and which are suitable for conducting a simplified dynamic SSI analysis, whether it is a specialised software for SSI analysis (PLAXIS, FLAC, COMSOL, GEFDYN, FLUSH, SASSI, HOPDYNE) or a general software (ANSYS, ADINA, ABAQUS, DYNAFLOW). (All the mentioned software can also perform a full dynamic analysis, in which the seismically induced increase in force in the tunnel lining is directly obtained as the output quantity of the corresponding numerical model selected for the simulation of the shaking of the coupled tunnel–ground system; in addition to the acceleration time history and site characteristics, kinematic and dynamic interactions are also taken into account).

3. PROPERTIES OF THE GROUND AND THE TUNNEL IN THE PRESENT STUDY

The circular cross-section tunnel structure was considered to be placed in a soil layer of 30 m in thickness, which lay over a relatively stiff bedrock. An outer tunnel radius of 3.0 m was considered, while the lining thickness was 0.3 m. The overburden depth was 12 m and a centre of the tunnel was at the depth of 15 m.

The physical properties of the tunnel lining and the soil material surrounding the tunnel are shown in Figure 5. Given that the effects of soil–structure interaction depend on the relationship between the stiffness of the soil and the lining, in the present study a stiff soil is considered a soil in good condition, while soft saturated soil is used as an example of soil in poor condition.

The shear wave velocity profiles $C_S(z)$ are depicted in Figure 5 by vertical lines (solid line in the case of stiff soil in good condition (Fig. 5(a)) and dashed line in the case of soft saturated soil in poor condition (Fig. 5(b)). These lines present the so-called "equivalent velocity", which is the mean value of the soil shear wave velocity, needed to perform a one-dimensional seismic site response linear analysis. In the considered cases, an average shear wave velocity profile of 250 m/s for stiff soil and 110 m/s for soft soil was used in the study.

With regard to the soil shear modulus, it is in a linear analysis of a constant nature with respect to a constant value of the shear wave velocity. The modulus value in the case of stiff soil was $G_{gr} = G_{max} = 120$ MPa and for soft soil it was $G_{gr} = G_{max} = 21$ MPa (after Eq. (4)). The value of the damping coefficient is also constant given the linear analysis, and $D_{gr} = D_0 = 1\%$ was taken for stiff soil deposit, whereas $D_{gr} = D_0 = 2.5\%$ was considered for soft soil material.

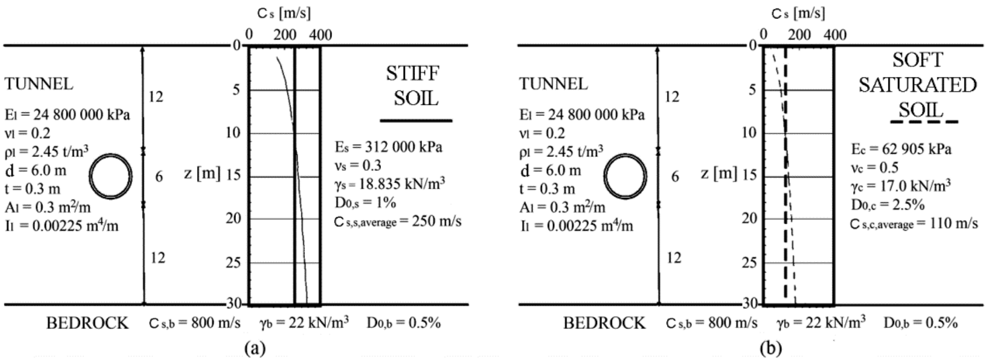

Figure 5. *Properties of the tunnel and the soil: (a) stiff soil; (b) soft saturated soil [27]*

4. COMPARISON OF ANALYTICAL AND NUMERICAL ANALYSIS RESULTS WITH REGARD TO DESIGN SOIL SHEAR STRAIN EVALUATION IN SEISMIC ANALYSIS OF TUNNELS

In the following study, both pseudo-static and simplified dynamic analysis methods were considered for the evaluation of the relevant (design) soil shear strain in seismic analysis of tunnels, regarding the idealised tunnel geometry and ground properties. In the quasi-static approach, the soil shear strain induced by shear body waves was calculated using the most frequently applied expressions, whereas in the simplified dynamic analysis the soil shearing was determined by performing a one-dimensional free-field ground response analysis in the corresponding programme. A comparison of the results of these two approaches, considering both good and poor soil conditions with linear elastic behaviour assumption, is performed, and the significant mutual differences are evaluated.

4.1. ONE-DIMENSIONAL SSR LINEAR ANALYSIS

The SSR analysis was performed using the programme EERA (Equivalent-linear Earthquake site Response Analysis) [20], which is integrated with the MS-Excel spreadsheet programme.

This programme allows performing 1D linear or equivalent linear SSR analyses, considering horizontally layered subsoils with vertical propagation of horizontal shear waves (SH-waves). The behaviour of the horizontal soil layer is simulated by the Kelvin–Voigt solid, with shear modulus and viscous damping characterising the properties of soil layers. Solving the wave propagation equations is done in the frequency domain (FD).

In the EERA programme, the bedrock can be simulated as rigid (by selecting the option "inside"), or as elastic (by selecting the option "extract", which assigns its properties to the last soil layer). For the sake of transforming the signal from the outcropping rock to the bedrock placed at the bottom of the soil layer, an appropriate transfer function is applied to the input signal, thus taking into account the transfer of shear stress between the bedrock and the overlying layer [18].

In these analyses, the soil conditions and soil behaviour were modelled in accordance with Figure 5. The seismically induced free-field soil deformations were calculated assuming that the soil behaviour is linearly elastic, and consequently, the soil shear modulus and damping coefficient are constant and do not depend on the shear level during the analysis.

In this study, the time history acceleration record of the 1995 Kobe Earthquake in Japan was considered, as this earthquake was the most destructive event for underground infrastructure in recorded history. In view of the fact that there is no recorded acceleration of strong ground motion at the depths where the tunnels are being built, the existing accelerogram recorded on the free surface was scaled to 0.25 g, thus accounting for the attenuation of the strong ground motion with depth [18]. The acceleration time history used in the SSR analyses (magnitude *Mw* = 6.5, distance from source to site = 26.4 km) is illustrated in Figure 6. The maximum value of the input acceleration time history was 0.251 g (2.46 m/s^2) and it occurred approximately 7.3 s after the start of earthquake excitation. The given earthquake acceleration input was applied to the bottom boundary of the soil model, whereby the bedrock was simulated as rigid by choosing the option "inside".

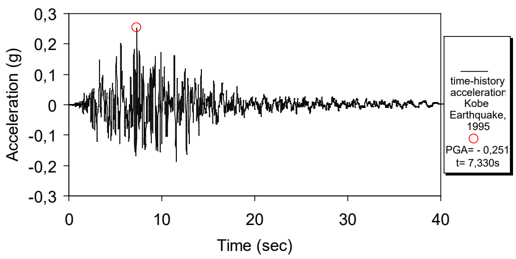

Figure 6. *Scaled accelerogram of the 1995 Kobe Earthquake in Japan used in the study [27]*

Analyses of the soil response to seismic motion enabled the calculation of the maximum values of acceleration and shear strains in the soil for the both considered cases, stiff soil and soft saturated soil, which is shown in the following diagrams (Fig. 7). With regard to Figure 7(a), concerning the given input motion, maximum ground acceleration value for the case of soft soil deposit is 1.45g, whereas in case of soft saturated soil peak ground acceleration is 0.76g. Consequently, the stiff soil column resulted in a higher peak ground acceleration compared to the value obtained for the saturated soft soil, in which case the accelerations were considerably lower along the depth of the soil column. This is in agreement with the property of soft saturated soil in terms of a higher damping ratio due to worse soil conditions, and therefore the soil ability to absorb more of the energy of the seismic wave, which ultimately results in significantly lower ground acceleration values.

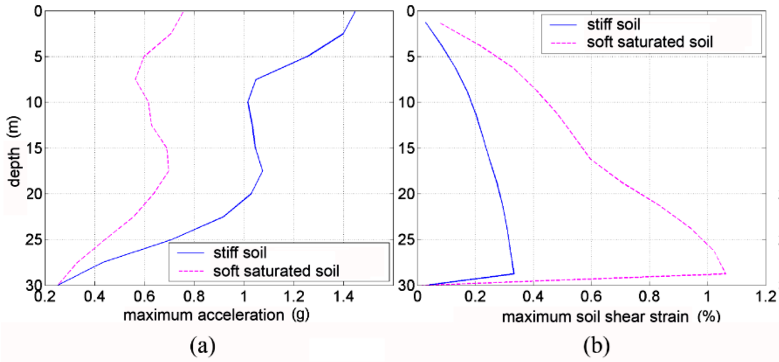

Figure 7 . *Profiles for stiff soil and soft saturated soil: (a) maximum acceleration; (b) maximum soil shear strain [27]*

4.2. COMPARATIVE STUDY OF ANALYTICAL AND NUMERICAL RESULTS

4.2.1. Evaluation of relevant (design) soil shear strain: pseudo-static approach

Parameters of the earthquake and soil:

- *Mw* = 6.5, distance from source to site = 26.4 km;
- Maximum ground particle acceleration at the free surface (stiff soil): $a_{S,max}$ = 1.45g;
- Maximum ground particle acceleration at the free surface (soft saturated soil): $a_{S,max}$ = 0.76g.

Stiff soil

Estimation of ground motion at the depth of the tunnel (according to Eq. (5) and Table 1):

$$a_{S,depth} = (\text{coefficient from Table 1}) \cdot a_{S,max} = 0.85 \cdot a_{S,max} = 0.85 \cdot 1.45\text{g} = 1.23\text{g}\,.$$

Determination of peak particle (peak ground) velocity at the depth of the tunnel (according to Eq. (6) and Table 2):

$$V_{S,depth} = (\text{value from Table 2}\,) \cdot a_{S,depth} = 102\,\frac{\frac{\text{cm}}{\text{s}}}{\text{g}} \cdot 1.23\text{g} = 125.5\,\frac{\text{cm}}{\text{s}} = 1.25\,\frac{\text{m}}{\text{s}}.$$

Computation of the relevant (design) soil shear strain (according to Eq. (8)):

$$\gamma_{rel} = \frac{V_{S,depth}}{C_S} = \frac{1.25\,\frac{\text{m}}{\text{s}}}{250\,\frac{\text{m}}{\text{s}}} = 0.005 = 0.5\%\,.$$

Soft saturated soil

Estimation of ground motion at the depth of the tunnel (according to Eq. (5) and Table 1):

$$a_{S,depth} = (\text{coefficient from Table 1}) \cdot a_{S,max} = 0.85 \cdot a_{S,max} = 0.85 \cdot 0.76\text{g} = 0.65\text{g}\,.$$

Determination of peak particle (peak ground) velocity at the depth of the tunnel (according to Eq. (6) and Table 2):

$$V_{S,depth} = (\text{value from Table 2}) \cdot a_{S,depth} = 132\,\frac{\frac{\text{cm}}{\text{s}}}{\text{g}} \cdot 0.65\text{g} = 85.8\,\frac{\text{cm}}{\text{s}} = 0.858\,\frac{\text{m}}{\text{s}}\,.$$

Computation of the relevant (design) soil shear strain (according to Eq. (8)):

$$\gamma_{rel} = \frac{V_{S,depth}}{C_S} = \frac{0.858\,\frac{\text{m}}{\text{s}}}{110\,\frac{\text{m}}{\text{s}}} = 0.0078 = 0.78\%\,.$$

4.2.2. Evaluation of relevant (design) soil shear strain: simplified dynamic approach

Linear EERA analysis revealed that for the considered stiff soil profile and input seismic data, ground acceleration value at the tunnel axis level is 1.03g. In case of soft saturated soil, ground acceleration at the tunnel spring line location is 0.64g (Fig. 7(a)). Accordingly, the numerically obtained values compare reasonably well with those computed by simplified expressions within pseudo-static approach.

The soil shear deformations are shown in Figure 7(b). Based on the given diagrams, the relevant value of soil shear deformation was also calculated, as the average value of soil shear deformation within the space that will be occupied by the tunnel structure, between the crown and the invert of the tunnel. The maximum value of soil shear deformation for the case of stiff soil deposit is 0.33%, whereas for the case of soft saturated soil it is 1.06%. The average value of the shear deformation of the soil at the location of the tunnel, i.e. in the range of depths between the tunnel crown and the invert, is 0.24% in stiff soil and 0.57% in soft saturated soil. Consequently, the shear deformations of the soil for the case of soft saturated soil are considerably higher, given the weaker properties of the soil in terms of the presence of water, higher damping values, and a higher share of seismic wave energy absorption.

4.2.3. Comparative analysis of the obtained results

The relevant (design) values of seismically induced soil shear strain, obtained by analytical (pseudo-static) approach and numerical (simplified dynamic) approach, considering both the stiff subsoil and the soft saturated subsoil are set side-by-side in Table 4.

Table 4. *Comparison of pseudo-static and simplified dynamic approaches for relevant soil shear strain evaluation concerning stiff soil and saturated soft soil deposits*

Soil	**Stiff soil**			**Soft saturated soil**		
approach / *relevant shear strain*	***Analytical (pseudo-static)***	***Numerical (simplified dynamic)***	***Analytical vs. Numerical***	***Analytical (pseudo-static)***	***Numerical (simplified dynamic)***	***Analytical vs. Numerical***
γ_{rel}	0.005	0.0024	52%	0.0078	0.0057	27%

According to the comparison of the obtained results, the conclusion that arises is that the pseudo-static analytical expressions generally provide a higher soil shear strain than that obtained by the simplified dynamic linear analysis. This conclusion holds particularly true for the case of stiff soil deposits, since the former approach yields prediction of the strain level closer to the simplified dynamic analysis for the case of soft soil deposit, thus implying that simplified relations based on quasi-static approach are in a better agreement with poorer soil properties, such as loose sand or water-saturated clay, which have lower values of the compressional and flexural stiffness.

It can be summarised that pseudo-static analysis can result as more conservative in estimating the strain level in comparison with the simplified dynamic analysis approach, thereby overestimating forces in the tunnel lining to an extent. However, from the aspect of engineering practice, simple analytical expressions are very useful and the results obtained in this way can be considered to be on the side of safety.

5. COMPARISON OF ANALYTICAL AND NUMERICAL ANALYSIS RESULTS WITH REGARD TO SEISMICALLY INDUCED INTERNAL LINING FORCES BASED ON SSI APPROACH

In order to assess the seismically induced tunnel–ground interaction effects, given the main soil classes, stiff and soft soils, an analysis was performed based on a comparison of the results obtained by a simplified analytical approach and a simplified numerical model. Based on that, the ability of the analytical and numerical models in simulating the most significant aspects of the interaction effects was evaluated, along with the most significant factors that affect the tunnel–ground interaction under an earthquake action.

5.1. DETERMINATION OF SEISMICALLY INDUCED TUNNEL LINING FORCES ACCORDING TO ANALYTICAL EXPRESSIONS

Firstly, on the basis of the previously presented analytical expressions for seismically induced tunnel lining forces that takes into account the kinematic tunnel–ground interaction effects, proposed by Wang [25], the maximum values of tunnel lining internal forces M_{max} and T_{max} were calculated for the case of no-slip condition.

5.1.1. Stiff soil

Based on Figures 4 and 5(a), and according to Eqs. (14), (13), (11), (12), (9), and (10), respectively:

Flexibility ratio:

$$F = \frac{E_{gr}(1-\nu_{lin}{}^2)r^3}{6E_{lin}I_{lin}(1+\nu_{gr})} = \frac{312000(1-0.2^2)3.0^3}{6\cdot 24.8\cdot 10^6\cdot 0.00225(1+0.3)} = 18.581\,.$$

Compressibility ratio:

$$C = \frac{E_{gr}(1-\nu_{lin}{}^2)r}{E_{lin}t_{lin}(1+\nu_{gr})(1-2\nu_{gr})} = \frac{312000(1-0.2^2)3.0}{24.8\cdot 10^6\cdot 0.3(1+0.3)(1-2\cdot 0.3)} = 0.232\,.$$

Moment response coefficient K_1:

$$K_1 = \frac{12(1-\nu_{gr})}{2F+5-6\nu_{gr}} = \frac{12(1-0.3)}{2\cdot 18.581+5-6\cdot 0.3} = 0.208\,.$$

Thrust response coefficient K_2:

$$K_2 = 1+\frac{18.581[(1-2\cdot 0.3)-(1-2\cdot 0.3)0.232]-\frac{1}{2}(1-2\cdot 0.3)^2+2}{18.581[(3-2\cdot 0.3)+(1-2\cdot 0.3)0.232]+0.232\left(\frac{5}{2}-8\cdot 0.3+6\cdot 0.3^2\right)^2+6-8\cdot 0.3} = 1.152\,.$$

Maximum moment due to S-waves (for the relevant (design) value of earthquake induced soil shear strain obtained by 1D SSR analysis ($\gamma_{rel} = 0.0024$)):

$$M_{max} = \frac{1}{6}K_1\frac{E_{gr}}{1+\nu_{gr}}r^2\cdot\gamma_{rel} = \frac{1}{6}0.208\frac{312000}{1+0.3}3.0^2\cdot 0.0024 = 179.8\text{ kNm}\,.$$

Maximum tangential thrust due to S-waves (for the relevant (design) value of earthquake induced soil shear strain obtained by 1D SSR analysis ($\gamma_{rel} = 0.0024$)):

$$T_{max} = K_2\frac{E_{gr}}{2(1+\nu_{gr})}r\cdot\gamma_{rel} = 1.152\frac{312000}{2(1+0.3)}3.0\cdot 0.0024 = 995.6\text{ kN}\,.$$

5.1.2. Soft saturated soil

Based on Figures 4 and 5(a), and according to Eqs. (14), (13), (11), (12), (9), and (10), respectively:

Flexibility ratio:

$$F = \frac{E_{gr}(1-\nu_{lin}{}^2)r^3}{6E_{lin}I_{lin}(1+\nu_{gr})} = \frac{62905(1-0.2^2)3.0^3}{6\cdot 24.8\cdot 10^6\cdot 0.00225(1+0.5)} = 3.247\,.$$

Compressibility ratio (in Eq. (14), the Poisson's ratio was set to 0.49, because a value of 0.5 will result in an infinite value of the ratio):

$$C = \frac{E_{gr}(1-\nu_{lin}{}^2)r}{E_{lin}t_{lin}(1+\nu_{gr})(1-2\nu_{gr})} = \frac{62905(1-0.2^2)3.0}{24.8\cdot 10^6\cdot 0.3(1+0.49)(1-2\cdot 0.49)} = 0.812\,.$$

Moment response coefficient K_1:

$$K_1 = \frac{12(1-\nu_{gr})}{2F+5-6\nu_{gr}} = \frac{12(1-0.5)}{2\cdot 3.247+5-6\cdot 0.5} = 0.706\,.$$

Thrust response coefficient K_2:

$$K_2 = 1+\frac{3.247[(1-2\cdot 0.5)-(1-2\cdot 0.5)0.812]-\frac{1}{2}(1-2\cdot 0.5)^2+2}{3.24[(3-2\cdot 0.5)+(1-2\cdot 0.5)0.812]+0.812\left(\frac{5}{2}-8\cdot 0.5+6\cdot 0.5^2\right)^2+6-8\cdot 0.5} = 1.231\,.$$

Maximum moment due to S-waves (for the relevant (design) value of earthquake induced soil shear strain obtained by 1D SSR analysis ($\gamma_{rel} = 0.0057$)):

$$M_{max} = \frac{1}{6}K_1\frac{E_{gr}}{1+\nu_{gr}}r^2 \cdot \gamma_{rel} = \frac{1}{6}0.706\frac{62905}{1+0.5}3.0^2 \cdot 0.0057 = 252.6\ \mathrm{kNm}\,.$$

Maximum tangential thrust due to S-waves (for the relevant (design) value of earthquake induced soil shear strain obtained by 1D SSR analysis (γ_{rel} = 0.0057)):

$$T_{max} = K_2\frac{E_{gr}}{2\left(1+\nu_{gr}\right)}r \cdot \gamma_{rel} = 1.231\frac{62905}{2(1+0.5)}3.0 \cdot 0.0057 = 440.2\ \mathrm{kN}\,.$$

5.2. A SIMPLIFIED NUMERICAL FINITE ELEMENT MODEL

In the present study, a two-dimensional (2D) simplified dynamic linear analysis was carried out using the finite element (FE) based commercial software ANSYS [28], by employing a continuous FE model. The idealisations, on which the performed analysis was based, were as follows: (1) it was assumed that the soil surrounding the tunnel is a homogeneous, isotropic, elastic half-space; (2) it was assumed that the behaviour of the tunnel lining is linearly elastic; (3) two-dimensional analyses under the plane-strain condition were conducted, which separated the transverse response from the longitudinal response, thus assuming uniform properties of the soil and the tunnel structure along the length of the tunnel.

With an aim to minimise boundary effects, the soil was modelled in such a way that the outer boundaries extended a distance > 4*d* (*d* being the tunnel diameter). Therefore, the width of the mesh was selected to be 54 m, whereas its height was 30 m, which is in accordance with the thickness of the soil deposit overlying the bedrock. The ground was modelled by plane-strain solid elements with two degrees of freedom (U_x, U_y) at each node, whereas the tunnel was modelled by beam elements with three degrees of freedom (U_x, U_y, ROT_z). The FE mesh consisted of 368 triangular solid elements with six nodes and 36 beam elements with two nodes. The ANSYS free-meshing algorithm was used, along with mesh refinement in the vicinity of the tunnel. To simulate the no-slip condition, the tied degrees-of-freedom boundary condition was applied along the joint surface of the tunnel lining and the surrounding ground, thereby assuming the compatibility of the lining and ground displacements and constraining the nodes on the two sides of the different meshes to deform identically [29]. Displacements in the vertical and horizontal directions were fixed at the bottom of the FE mesh, thus modelling rigid bedrock beneath the soil deposit. The upper horizontal boundary of the FE model, which simulated the ground surface, was considered free.

All 2D simplified dynamic analyses presented herein were preceded by static analyses to verify the model under static conditions as well. In performing static analyses, on the one hand, in order to restrict horizontal displacements along the vertical boundaries of the model, supports in the form of rollers were used. In performing dynamic analyses, on the other hand, the vertical displacements were constrained along the lateral boundaries of the model. The seismic loading was simulated under simple shear conditions, obtained by means of 1D SSR analysis in the code EERA. Even though such simplified approaches cannot adequately simulate the variations of soil stiffness and strength that occur during an earthquake and do not take into account any dynamic tunnel–ground interaction effects, however, they usually provide a reasonable assessment of the earthquake load.

Considering the stiff soil, the maximum calculated values of displacements at the tunnel section were 2.43 cm at the level of the crown of the tunnel and 1.67 cm at the level of the tunnel invert. Accordingly, the relative displacement between the crown and the invert of the circular tunnel cross-section is of lower value (0.76 cm), which led to less distortion (ovalisation) of the circular cross-section of the tunnel. In the case of soft saturated soil, as a result of significantly higher soil shear strain values, larger soil displacements occured (12.73 cm and 10.19 cm at the top and bottom of the tunnel, respectively), which imposed a larger relative displacement between the tunnel crown and invert (2.54 cm) compared to the stiff-soil case, and thus resulted in significantly greater ovalisation of the tunnel structure (Fig. 8).

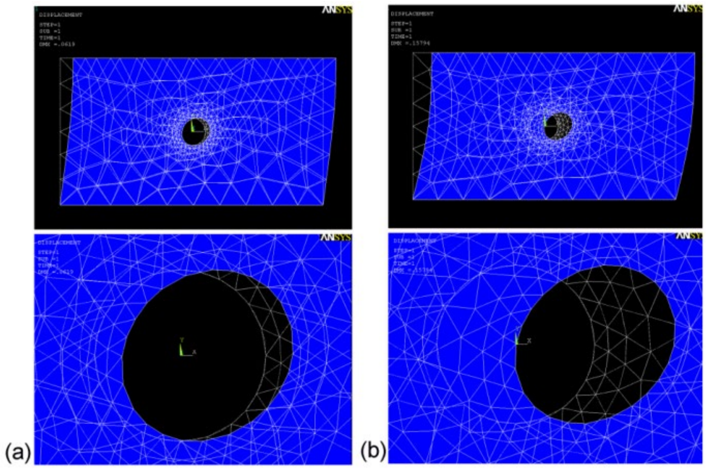

Figure 8. *Seismically induced ovalisation of the circular tunnel cross-section (displacements enlarged 25 times): (a) stiff soil; (b) soft saturated soil [27]*

5.3. COMPARISON OF THE ANALYTICAL AND NUMERICAL RESULTS

The obtained numerical results were compared to the analytical solutions, as presented by diagrams in Figure 9. Thrust (*T*) and bending moment (*M*) were determined in terms of the angle θ, which was measured counterclockwise with respect to the axis of the tunnel spring line.

Based on the obtained analytical and numerical results with regard to soils of good and poor conditions, it was observed that the magnitude of the thrust *T* has a much stronger effect on the stresses in the tunnel lining when compared to the bending moment *M*, which is typical for the no-slip assumption considered in the given analysis and in line with the findings of Hashash et al. [16].

When considering the distribution of thrust in the tunnel lining, it could be seen that the numerical results related to the soft saturated soil agree quite well with the Wang's analytical solution, whereas in the case of the stiff soil, the numerically obtained accumulated thrust provides a fairly consistent distribution pattern with, however, somewhat lower maximum values than those of the Wang's solution.

In terms of seismically induced bending moments, the numerical model accounting for the soft saturated soil predicted a distribution that conforms that obtained by the Wang's analytical approach. In the case of stiff soil, however, the numerically obtained distribution is quite similar to that determined by the Wang's analytical solution, with slightly lower maximum values.

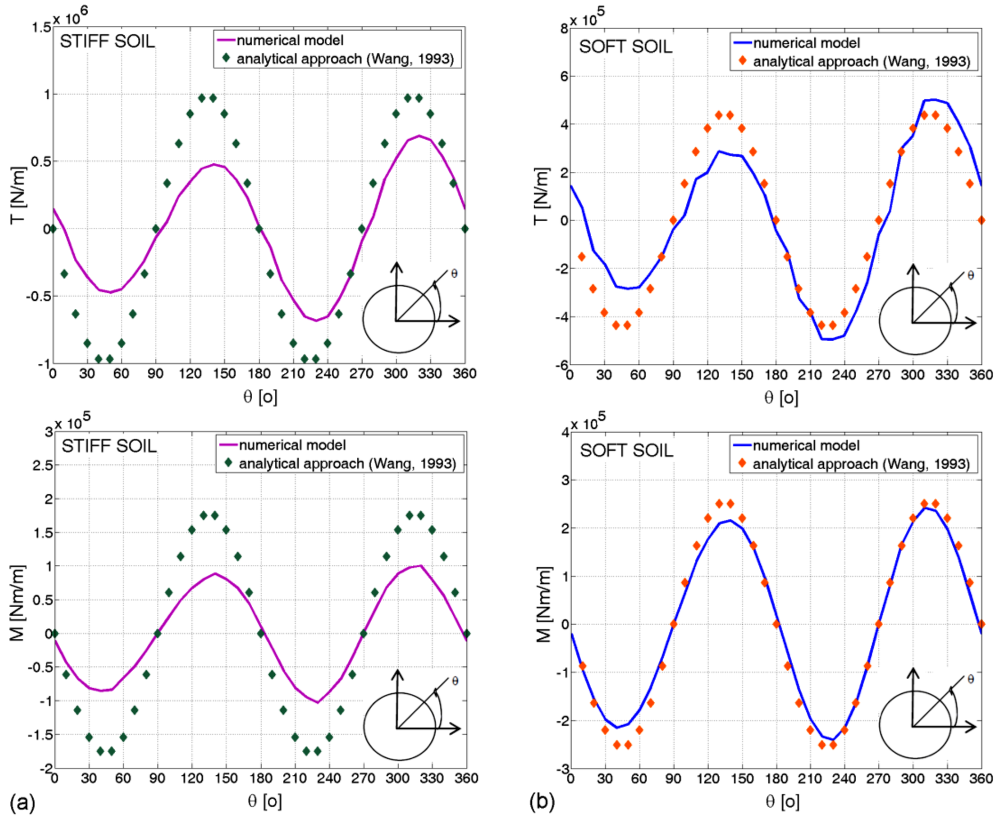

Figure 9. *Comparison of the analytical and numerical results: (a) stiff soil; (b) soft saturated soil [27]*

The maximum values of internal tunnel lining forces caused by the earthquake, obtained by the analytical and numerical models for the case of stiff soil and the case of soft saturated soil, are highlighted in Table 5.

Table 5. *Comparison of the analytical and numerical results related to the stiff soil and soft soil*

Soil	**Stiff soil**			**Soft saturated soil**		
approach / *force*	***Analytical (pseudo-static)***	***Numerical (simplified dynamic)***	***Analytical vs. Numerical***	***Analytical (pseudo-static)***	***Numerical (simplified dynamic)***	***Analytical vs. Numerical***
M_{max} **[kNm/m]**	180	103	≈ 40%	253	242	≈ 4%
T_{max} **[kN/m]**	996	687	≈ 30%	440	500	≈ 10%

A common conclusion that can be drawn from the results obtained according to the frequently used simplified analytical approach according to Wang and performed two-dimensional simplified dynamic linear finite element analysis is that Wang's analytical expressions more faithfully simulate soils of poorer properties and lower stiffness, such as, for example, loose sand or soft undrained clay.

It can also be summarised that pseudo-static analysis approach may give a more conservative assessment of internal tunnel lining forces compared to the simplified dynamic analysis approach. Here again, from the aspect of engineering practice, simple analytical expressions are very useful and the results obtained in this way can be considered to be on the side of safety.

Given the results shown above, a difference between the seismically induced internal tunnel lining forces for the case of stiff soil in relation to the case of soft saturated soil can be seen, which clearly implies the significance of the tunnel–ground interaction effects.

Axial forces (thrust) in the case of stiff soil deposit are of higher values compared to the case of soft saturated soil. This results from significantly higher values of soil shear stress, due to the fact that stiff soil has better characteristics, and therefore higher compressional and flexural stiffness, which results in lower internal tunnel lining forces. This finding is consistent with the observation of Hashash et al. [16], according to which earthquake-induced tunnel lining forces increase with a decrease in the compressibility and flexibility ratio of the soil in relation to the lining.

On the other hand, regarding the distribution of the bending moment, the results of both analytical and numerical approaches showed that the moment values are considerably higher in the case of soft saturated soil, which is affected by significantly weaker properties and lower shear stiffness of the soil, leading to larger soil shear deformations, and therefore to larger seismically induced soil displacements.

6. CONCLUDING REMARKS

The tunnel–ground interaction effects have recently become an indispensable part of the analysis and design of tunnels under earthquake conditions, as these effects between the structure and the surrounding soil may result in higher pressures acting on the structure. The tunnel basically reacts to soil deformations, where the level of tunnel deformation depends primarily on the ratio of the tunnel lining stiffness and the soil stiffness. In the seismic analysis of tunnel structures, the peak ground strain during an earthquake is not relevant for the appearance of pressures on the tunnel structure, but the soil shear strain that occurs in the range of depths that correspond to the tunnel crown and invert.

The present study considered the most frequently used simple analytical expressions, regarding the idealised tunnel geometry and ground properties. The presented analytical expressions refer to the calculation of the relevant (design) soil shear strain that occurs in the range of depths that correspond to the tunnel crown and invert, on the one hand, and of the seismically induced forces in the tunnel lining considering the tunnel–ground interaction effects, on the other hand. The latter, proposed by Wang, are presented as one of the still most commonly used analytical expressions today. Furthermore, in order to evaluate the ability of the analytical expressions to simulate the most important aspects of the seismic performance of tunnels, numerical analyses were also carried out by one-dimensional free-field ground response analysis in the programme EERA and by the simplified dynamic soil–structure interaction analysis in the software ANSYS, respectively. Lastly, the results obtained by the simple analytical and numerical approaches were evaluated, given the main soil classes, stiff and soft soils.

Based on the comprehensive comparison of the obtained results, the following most significant conclusions could be drawn:

- The pseudo-static analytical expressions generally provide a higher soil shear strain than that obtained by the 1D SSR linear analysis. This conclusion holds particularly true for the case of stiff soil deposits, since the former approach yields prediction of the strain level closer to the simplified dynamic analysis for the case of soft soil deposit. This finding implies that simplified relations based on quasi-static approach are in a better agreement with poorer soil properties, which have lower extensional and flexural stiffness values.
- A common conclusion that can be drawn from the results obtained according to the frequently used simplified analytical approach according to Wang and the performed two-dimensional simplified dynamic linear finite element analysis is that the Wang's analytical expressions more faithfully simulate soils of weaker properties and lower stiffness, such as, for example, loose sand or soft undrained clay.
- The pseudo-static analytical expressions proved to be more conservative in estimating the strain level in comparison with the simplified dynamic analysis approach, thereby overestimating forces in the tunnel lining to an extent.
- In addition, the simplified analytical approach according to Wang resulted in a more conservative assessment of tunnel lining internal forces compared to the simplified dynamic analysis approach.
- It can be summarised that, although simple analytical expressions, considering both the design value of the soil shear strain and the seismically induced internal lining forces, are shown to be conservative, they are very fruitful as they give rational results from the aspect of engineering practice, which are on the side of safety.

In the examined case study, soil is assumed to behave in the linear elastic manner. Soil, however, rarely behaves this way. A more accurate approach would consider the nonlinear behaviour of the soil, by which damping and attenuation of the soil material will be taken into consideration. Moreover, the interface between the lining and the surrounding soil can be taken as a partial or full slip condition, which is particularly adequate in the case of an earthquake excitation of high frequency, as well as in the case of shallow-embedded tunnels. Finally, the presented analyses do not take into consideration the nonlinear behaviour of the tunnel lining and the possible cracking of the lining, so the assessed internal lining forces may differ somewhat from the forces actually acting in the lining.

However, when it comes to analytical expressions suitable from the aspect of engineering practice, a very serious and challenging task lies ahead, because, on the one hand, they should be as realistic as possible and include as many relevant parameters as possible, whereas, on the other hand, they should remain sufficiently simple and understandable for a design engineer for whom they are primarily intended.

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О УПРОШЋЕНИМ ПРИСТУПИМА СЕИЗМИЧКЕ АНАЛИЗЕ ТУНЕЛА

Сажетак: Прегледом актуелних сеизмичких стандарда за пројектовање тунела у свету и код нас утврђено је да, упркос значајном напретку у области принципа сеизмичке анализе тунела у последњих неколико деценија, чак и у најразвијенијим земљама још увек постоји недостатак систематских и прецизно утврђених правила сеизмичког пројектовања тунела. У овом раду размотрени су једноставни аналитички изрази, који се базирају на претпоставци идеализоване геометрије тунела и својстава тла, за одређивање меродавне вредности смичуће деформације тла која се јавља на делу између тунелског свода и инверта, са једне стране, и за срачунавање сеизмички индукованих сила у тунелској облози узимајући у обзир ефекте интеракције конструкције и тла, са друге стране. Такође, са циљем оцене аналитичких израза у погледу сагледавања најважнијих аспеката сеизмичког одговора тунела, спроведене су и нумеричке анализе једнодимензионалном анализом сеизмичког одзива тла у програму EERA и упрошћеном динамичком анализом интеракције тло–конструкција у софтверу ANSYS, респективно. На крају, извршено је поређење резултата добијених упрошћеним аналитичким и нумеричким приступима, уз разматрање два карактеристична случаја тла, чврстог тла добрих карактеристика и меког засићеног тла слабих карактеристика.

Кључне ријечи: кружни тунел, земљотрес, стандарди, сеизмичка анализа, упрошћени приступи